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# Stochastic memory process and its application to cumulative outage time in nuclear power plants

Mohamad Ali Azarm  
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**Azarm, Mohamad Ali, Ph.D.**

**Iowa State University, 1989**

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Stochastic memory process and its application to  
cumulative outage time in nuclear power plants

by

Mohamad Ali Azarm

A Dissertation Submitted to the  
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1989

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## 1. INTRODUCTION

Technical specifications are an intimate part of the regulatory framework. As required by 10CFR50.36<sup>1</sup>, plant technical specifications (TS) for power reactors are to include: 1) safety limits and limiting safety system settings, 2) limiting conditions of operation (LCO), 3) surveillance requirements (SR), 4) design features, and 5) administrative controls. NRC has developed and required, on a forward-fit basis, the use of standard technical specifications (STS) since 1975. This is done to make the TS (which applicants for operating licenses are required to utilize) more effective and efficient.

NUREG-1024<sup>2</sup>, "Technical Specifications - Enhancing the Safety Impact", has documented past experiences which indicate that lack of guidance on TS can affect both licensing and operations. This report found probabilistic methodologies to be useful in the decision-making process for establishing test intervals (TI) and allowed outage times (AOT) for safety components. As has been emphasized in NUREG-1050<sup>3</sup>, probabilistic assessments of nuclear power plant risks are valid inputs to decisions about whether to relax or further restrict regulatory requirements for the plant.

The Procedures for Evaluating Technical Specifications (PETS) Program was initiated in January 1984 at Brookhaven National Laboratory to examine approaches for developing and demonstrating a quantitative, reliability-based, decision-making process in evaluating TS. The program

was initially scoped to focus on two aspects of TS, viz., AOT and surveillance test intervals (STI).

During the course of program execution, as described in the original PETS Program Plan<sup>4</sup>, it was found that the current policy of establishing AOT may not be effective in controlling the plant risk. The AOT is conventionally defined as the maximum allowable continuous downtime for a safety component without the need for reactor shutdown. Therefore, a safety component can accumulate downtime almost indefinitely as long as the component is brought up to an operational state after each single outage within its prescribed AOT. In other words, the existing AOTs only control the risk of a single outage of a safety component with no control on its cumulative outage or the frequency with which the outages occur. Therefore, it was recommended by the PETS Program that the technical specification should be upgraded by providing a means to control the cumulative downtime of safety components.

There are various technical and administrative issues associated with establishing the allowable cumulative outage times (ACOT) that are to be resolved before such a regulatory change can take place. The identification and the resolution of the technical issues are investigated in this dissertation. The issues associated with the administrative and regulatory implementations will not be discussed here. The major technical issues are:

1. The advantages of ACOT over alternative policies:

One alternative policy to the ACOT approach is to control both the frequency of downtime and the allowable outage time per

downtime. In return, this would effectively control the plant risk induced by equipment outages. Therefore, it is required to show that a policy based on ACOT is more effective in controlling the cumulative downtime (therefore assuring safety) of a safety component without creating unnecessary restrictions on its operation compared to any other alternative policy.

2. The determination of the cumulative downtime probability distribution functions and their statistical characteristics:

To assure successful application of ACOT-based regulatory policy, it is required to evaluate the percentage of time that the ACOT requirement is violated due to the statistical nature of the process rather than the substandard performance of a component. To limit and control the number of violations, especially when a large number of nuclear power plants are involved, the probability distribution functions (PDF) for cumulative downtimes must be estimated with high accuracy under different scenarios.

3. The estimation of plant-risk reduction using ACOT-based regulation:

The main justification for any regulatory requirement is its risk reduction worth. Therefore, it is required to translate the ACOT limits to associated limits on plant risk. This aspect of the problem, even though not completely resolved, will be addressed within the framework of conventional probabilistic risk assessment methodologies.

4. The technical feasibility for extending ACOT to plant systems:  
The ACOT policies discussed thus far are expected to be applied to individual safety equipment. A natural extension to this process is to develop an overall ACOT for a safety system in a nuclear power plant. The probabilistic techniques for evaluating a system level ACOT probability distribution function and its ability to control plant risk are to be developed.

An attempt has been made in this thesis to construct a model to evaluate the concept of ACOT and its risk impact. The underlying models developed here are of general nature and can be applied in other areas where the analyst is faced with a situation where cumulative characteristics of a stochastic process are to be evaluated. A summary discussion of pertinent literature is given in Chapter 2. Chapter 3 provides the underlying models developed for evaluating the characteristics of a cumulative stochastic process in the context of ACOT. The various analytical and approximate solutions to these models are discussed in Chapter 4. Chapter 5 describes the development of a statistical package, namely the ACOT program, for evaluating the statistical characteristics of a cumulative process and compares it with various solution techniques. Chapter 6 provides the actual application of the ACOT program for the determination of allowable cumulative outage times for several safety components in nuclear power plants. Finally, the summary and conclusions of this study and recommendations for the extension of the methodologies are discussed in Chapters 7 and 8, respectively.

## 2. LITERATURE REVIEW

In this section, the pertinent literature for determination of cumulative downtime of a single component and of a system composed of many components is discussed. The general methodological approach used in this study is based on either alternating renewal processes or extensions of Markov processes. In addition to the literature concerning these two topics, some references deal with methods of solution of the equations derived for these two processes.

The concept of alternating renewal processes and the equations for the cumulative uptime distribution are discussed by Gnedenko, Belyayev, and Solovyev<sup>5</sup>. The same topic and similar equations are described by Barlow and Porchan<sup>6</sup>. Under the assumption of constant occurrence rate, namely exponential uptime survival distribution, Cinlar<sup>7</sup> provides similar equations for calculating the moments of uptime distribution for an alternating renewal process. This standard technique is known as the compound Poisson process which uses the concept of moment generating functions and has extensive application in reliability areas.

There are a large number of papers available that deal with the description and application of Markov processes. It is traditional to conceive of a stochastic process as defined by the totality of joint distribution functions. However, instead of defining the process by the joint probabilities, it is easier to use the conditional probability of the realization of a process at time  $t$  given the past history of the process. If this conditional probability is independent of all knowl-

edge prior to the next preceding instant, the process is considered as a Markov Process. The classical example of a stochastic process and the knowledge prior to the next preceding instant, the process is considered as a Markov Process. The classical example of a stochastic process and the use of Markov model is the displacement of a Brownian particle as detailed by E.G.D. Cohen<sup>8</sup>. An extension to Markov processes, known as semi-Markov models, takes into account the dependence between the transition parameters and the period of time spent in the last state before transition to the present state is made. Semi-Markov Processes have also been the subject of many studies. The mathematical models of Markov and semi-Markov processes with several applications to various areas of reliability modeling are discussed by R.A. Howard<sup>9</sup>. The applications of semi-Markov models both in discrete and continuous form for evaluation of system reliability are discussed by Malaiya<sup>10</sup> and Branson and Shah<sup>11</sup>.

Three other topics were also considered for literature review. These topics deal with methods of solutions for the equations that are used for determination of cumulative downtime distribution: namely Laguerre expansion, a stratified sampling code, and approximate evaluations of multiple convolutions.

The Laguerre transform, introduced by Keilson and Nunn<sup>12</sup> and further studied by Keilson, Nunn and Sumita<sup>13</sup> provides an algorithmic framework for the computer evaluation of multiple convolutions and other continuum operations. A specific application of Laguerre expansion, namely estimating the Lagrangeans of the maximum entropy function is developed in this study.

Any simulation code which samples from distributions of random variables is logically equivalent to a statistical experiment, and its output must be interpreted in the framework of statistics. The Monte Carlo technique is frequently used to evaluate the output of an inherently random process. Stratified sampling techniques are commonly used to assure that the simulation has covered all possible ranges of interest with a limited number of samplings and at the same time reduced the variance around the estimates being made for the expected values of outputs. Among the various sampling procedures available, three are most common: simple random sampling or crude Monte Carlo (CMC), factorial stratified sampling (FSS), and latin hypercube sampling (LHS). These three statistical sampling techniques are discussed in detail by McKay, Conover, and Whiteman<sup>14</sup>. A Fortran 77 program and User's Guide for the generation of LHS is developed by Iman and Shortencarier<sup>15</sup>. In addition to these three statistical sampling techniques, a superior sampling technique known as unified stratified sampling (USS) was introduced and compared to other statistical sampling techniques by Filshtein, Goldstein and Kozmin<sup>16</sup>. This sampling technique usually needs fewer samples than the other three techniques without loss of accuracy. However, this sampling technique has not been widely used in reliability areas.

The last topic for literature review deals with determining an approximate solution to the distribution of a sum of random variables. It is well known that the distribution of a sum of random variables can be written in the form of multiple convolutions and approaches a normal

distribution when the number of variables is large (central limit theorem). In cases where the number of random variables is limited, approximate solutions can be established taking advantage of the central limit theorem. These approximate solutions (relationships) are commonly known as the local limit theorem and can be found in any statistics book.

One approach to the problem of near normality is to make small corrections to the normal distribution approximation by using asymptotic expansions (Edgeworth or Gram Charlier) based on the central limit theorem. This technique, also known as the method of cumulants, has been widely used in the power-system literature<sup>17</sup>. The Edgeworth approximation is especially useful when the basic random variables are close to normality. Thus, when a random variable markedly departs from normality, the Edgeworth approximation turns out to be weak in estimating its distribution. A large amount of empirical evidence (Beard, Pentikainen and Pesonen<sup>18</sup>) also exists to show that the Edgeworth expansions are ineffective in estimating the tail probabilities beyond the  $2\sigma$  range.

The other approach to the problem of near normality is the use of Esscher's large deviation technique. In this case, the analyst is only interested in finding an accurate approximate solution to the part of the distribution rather than the whole distribution. The idea is to change the variables to induce a displacement in distribution such that the part of distribution which is of concern shifts to the central part where the local limit theorem or the Edgeworth approximation fits the

best. This intuitively plausible expectation can be formally justified for large systems by way of the Berry-Esseen Theorem<sup>19</sup>. An application of this method for computation of power-generating system reliability is reported by Mazumdar and Gaver<sup>20</sup>.

Several other large-deviation procedures for approximating the areas under the tail of the probability density function of a convolution have been described in the literature (see Helstrom<sup>21</sup>, 1978 for a list of references on this subject). These procedures consist of inverting the moment-generating function by the method of steepest descents to yield a satisfactory approximation to the probability density function of a convolution. This gives an asymptotic expansion whose dominant term is called the saddle point approximation. These techniques were not reviewed as part of this dissertation effort.

### 3. DEVELOPMENT OF MATHEMATICAL MODELS

A standby or operating component goes through a cyclic pattern of being up, or in an operable state, and then being down, or in a failed state. The component can be down, usually as a result of preventive maintenance or sometimes for a repair of a faulty (failed) part. The cumulative downtime in some period of time is the sum of the individual downtime durations in that time period. The probability distribution of the cumulative downtime depends upon the frequency at which the component is brought down and the distribution of the individual downtime durations (repair times). These two parameters are considered here to be random variables, governed by specific statistical behavior (probability distribution functions).

In nuclear power plants, the cumulative downtime generates a cumulative downtime risk for the component. When the component is down then there is an associated downtime risk due to the component being out of service. In other industries, the cumulative downtime risk may be determined in terms of loss of production or cost. It is reasonable to assume that the cumulative downtime risk is proportional to cumulative downtime period.

There are many analogies to the problem of cumulative downtime and cumulative downtime risk in other areas of engineering. Three of these problems are described briefly to illustrate the potential areas for application of the methodologies developed in this dissertation.

1. Mechanical component fatigue (cumulative shock model):

In this analogy, a mechanical component experiences a number of shocks or stresses that are generated by a random number of demands. For this problem, we assume that the number of shocks (demands) within a period and the associated stresses are random variables (in this problem, we may only be concerned about the stresses above a given value). The cumulative stress within a time period then is the analog to the cumulative downtime and the methodologies developed here can be used to estimate the component survival probability versus time.

2. Slowing down in a non-absorbing medium:

In this analogy, a beam of neutrons is passing through a wall of non-absorbing medium, therefore, the distribution of neutron energy at any point depends on the number of collisions and the distribution of energy loss per collision (change of lethargy). Both of these quantities are considered to be random variables and the cumulative energy loss as a result of scattering can be considered as the analog to cumulative downtime distribution.

3. Cumulative radiation defect on materials:

In this analogy, we consider a sample of material irradiated by a flux of neutrons with a given energy distribution. The resulting number of vacancies and interstitials induced by neutron collision with crystal atoms as a function of neutron energy can be considered as random variables. Therefore, the

cumulative number of defects (either interstitials or vacancies) can be considered to be an analog to cumulative downtime and its distribution can be determined.

The remainder of this chapter describes several mathematical models that can be used for estimating the cumulative downtime distribution of a single component as well as a system composed of several components. The various solution techniques for these models are discussed in Chapter 4.

### 3.1 Compound Poisson Process

It is a well known theorem that the sum of  $N$  independent random variables,  $S_n$ , defined by Eq. 1, has a probability distribution function given by Eq. 2:

$$S_n = x_1 + x_2 + \dots + x_n \quad , \quad (1)$$

$$g(S_n) = f(x_1) * f(x_2) * \dots * f(x_n) \quad , \quad (2)$$

where "\*" stands for the convolution operator and  $f(S_n) = w(S_n)$  (a Dirac delta function at  $S_n = 0$ ).

The compound Poisson process extends the above theorem to the case where the number of variables contributing to the sum is itself a random variable generated by a Poisson process. In this case, the probability distribution function for the sum of the random variables,  $s$ , is given by Eq. 3,

$$g(s) = \sum_{i=0}^{\infty} g_i(s) p(i) \quad , \quad (3)$$

where  $p(i)$  is the probability that exactly  $i$  random variables are summed.

If the  $x_i$  are independent and identically distributed (iid) random variables with probability distribution function  $f(x)$  and  $p(i)$  results from a Poisson process with parameter  $\lambda$ , then the above process is known as a compound Poisson process. In this case, the probability of exactly  $m$  occurrences of the random variables in a period  $t$  is given by Eq. 4;

$$P(m) = (\lambda t)^m e^{-\lambda t} / m! \quad \text{for all } m \in (0, \infty) \quad . \quad (4)$$

Therefore, Eq. 3 can be expressed in the form of Eq. 5

$$g(s) = \sum_{i=0}^{\infty} (\lambda t)^i e^{-\lambda t} / i! f^{(i)}(s) \quad , \quad (5)$$

where  $f^{(i)}(s)$  is the  $i$ -th convolution of  $f$ .

If both sides of Eq. 5 are multiplied by  $e^{i\sigma s}$  and integrated over all values of "s", where  $s$  belongs to  $(0, \infty)$ , Eq. 6 can be obtained;

$$G(\sigma) = \sum_{i=0}^{\infty} (\lambda t F(\sigma))^i / i! e^{-\lambda t} \quad , \quad (6)$$

where  $G(\sigma)$  and  $F(\sigma)$  are the characteristic functions of  $g(s)$  and  $f(s)$  respectively, and are defined by:

$$G(\sigma) = \int_0^{\infty} g(s) e^{i\sigma s} ds \quad , \quad (7)$$

$$F(\sigma) = \int_0^{\infty} f(s) e^{i\sigma s} ds \quad . \quad (8)$$

The above characteristic functions always exist since both  $f(s)$  and  $g(s)$  are probability distribution functions. Having the characteristic functions, the moments of the distributions, if they exist, can be writ-

ten in terms of the derivatives of their characteristic functions evaluated at  $\sigma=0$ :

$$M_f^k = -(i)^k \left. \frac{d^k F(\sigma)}{d\sigma} \right|_{\sigma=0} \quad (9)$$

Eq. 6 can be summed and written in a closed form, yielding Eq. 10:

$$G(\sigma) = e^{-\lambda t} (1-F(\sigma)) \quad (10)$$

Using the relationship between the moments and the characteristic function as given in Eq. 9, all the moments of  $g$  can be estimated using the known values for the moments of  $f$ . Table 1 shows the relationships for the first five moments of  $f$  and  $g$  distributions.

The compound process described here can be used for estimating the distribution of cumulative shocks of a component during a period when the shocks occur randomly based on a Poisson process. The methodology can be further extended to account for different types of shocks each with a different frequency of occurrence. In this case, Eq. 10 can be extended to Eq. 12:

$$G(\sigma) = \prod_{i=1}^n G_i(\sigma) \quad (12)$$

where  $G_i(\sigma)$  is the characteristic function for cumulative shocks of type  $i$ .

Since  $G_i(\sigma)$  has exponential form,  $G(\sigma)$  can be written in the form of Eq. 13

$$G(\sigma) = e^{-t \sum_{i=1}^n \lambda_i} (1-F_i(\sigma)) \quad (13)$$

Table 1. Moment Relations of g and f Derived from Compound Poisson Process

---


$$M_1^g = (\lambda t) \cdot M_1^f$$

$$M_2^g = (\lambda t) \cdot M_2^f + (\lambda t) \cdot M_1^f \cdot M_1^g$$

$$M_3^g = (\lambda t) \cdot M_3^f + 2 \cdot (\lambda t) \cdot M_2^f \cdot M_1^g + (\lambda t) \cdot M_1^f \cdot M_2^g$$

$$M_4^g = (\lambda t) \cdot M_4^f + 3 \cdot (\lambda t) \cdot M_3^f \cdot M_1^g + 3 \cdot (\lambda t) \cdot M_2^f \cdot M_2^g + M_1^f \cdot M_3^g \cdot (\lambda t)$$

$$M_5^g = (\lambda t) \cdot M_5^f + 4 \cdot (\lambda t) \cdot M_4^f \cdot M_1^g + 6 \cdot (\lambda t) \cdot M_3^f \cdot M_2^g + 4 \cdot (\lambda t) \cdot M_2^f \cdot M_3^g + (\lambda t) \cdot M_1^f \cdot M_4^g$$

General recursive equation:

$$M_j^g = \sum_{i=1}^j \binom{i-1}{j-1} M^f (j-(i-1)) \cdot m^g(i-1)$$

where  $M^f(i)$  stands for  $M_i^f$

---

Now, if we define

$$\Lambda = \sum_{i=1}^n \lambda_i \quad , \quad (14)$$

and

$$F(\sigma) = \sum_{i=1}^n \lambda_i F_i(\sigma) / \Lambda \quad (15)$$

Eq. 13 can be rewritten in terms of  $\Lambda$  and  $F$  in exactly the same format as Eq. 10 for which the moment estimation process was described.

Application of the compound Poisson process to the evaluation of cumulative downtime distribution is straightforward. Here, the downtime frequency of each component is assumed to follow an exponential distribution with occurrence rate  $\lambda_i$ , and the repair distribution,  $f$ , can take the form of any general probability distribution function. The cumulative downtime distribution (or its moments) evaluated by this method is slightly approximate due to the inherent assumption of the compound Poisson process, namely the independence of  $t$  and  $\sigma$ . The consequences of this assumption are:

1. Given a downtime of  $\sigma$  within a period  $t$  for a component, the total operable time (uptime) is considered to be  $(t-\sigma)$ . Therefore, application of the compound Poisson process to cumulative downtime is only justified when  $\sigma$  is much less than  $t$ , or in another word the value of  $t-\sigma$  can be approximated by  $t$ .
2. In cumulative downtime problems, the value of  $\sigma$  (cumulative downtime) is to be always less than  $t$ . Due to independence of  $t$  and  $\sigma$  in a compound Poisson process, this consideration is

not accounted for. Therefore, the application of the compound Poisson process to evaluation of cumulative downtime is only valid for cases when the probability of  $\sigma$  exceeding  $t$  is negligible.

Generally, when the expected value of  $\sigma$  is much smaller than the  $t$  value (ratio of expected  $\sigma$  to  $t$  less than  $1/30$ ), the compound Poisson process provides a reasonably good approximation to the exact solution.

### 3.2 Alternating Renewal Process

The alternating renewal process for a single component assumes that the individual uptime durations each have the same probability distribution and the individual downtime durations each have the same (separate) probability distribution. This implies that there are no systematic trends in time in the uptime and downtime durations. This assumption is also applicable to a compound Poisson process with the added restriction that the uptime durations are exponentially distributed. In an alternating renewal process, the probability distribution of the uptime durations and the probability distribution of the downtime durations are, in general, different and can be of any discrete or continuous form. Under these assumptions, the uptimes and downtimes form an alternating renewal process and standard renewal equations can be used to obtain the cumulative downtime distribution.

From Gnedenko, Belyayev, and Solovyev<sup>5</sup>, page 115, for an alternating renewal process, the equation for the cumulative uptime distribution can be expressed as

$$P(h_t < s) = \sum_{n=0}^{\infty} [G_n(t-s) - G_{n+1}(t-s)] F_{n+1}(s) \quad , \quad (16)$$

where

$P(h_t < s)$  = the probability that the cumulative uptime  $h_t$  in time period  $t$  is less than  $s$

$G_n(t-s)$  = the probability that the sum of  $n$  downtime durations is less than or equal to  $t-s$

$G_{n+1}(t-s)$  = the probability that the sum of  $n+1$  downtime durations is less than or equal to  $t-s$ , and

$F_{n+1}(s)$  = the probability that the sum of  $n+1$  uptime durations is less than or equal to  $s$ .

The distribution  $G_n(t-s)$  is the  $n$ th convolution of the downtime distribution  $G(x)$ ; each downtime duration is assumed to have the same distribution  $G(x)$ . The distribution  $G_{n+1}(t-s)$  is the  $(n+1)$ th convolution. The distribution  $F_{n+1}(s)$  is the  $(n+1)$ th convolution of the uptime distribution of  $F(x)$ ; each uptime duration is assumed to have the same distribution  $F(x)$ . The convolutions are started with  $F_0(x) = 1$  and  $G_0(x) = 1$  for all values of  $x$ .

Since the cumulative downtime is simply  $t$  minus the cumulative uptime, Eq. 16 also provides the cumulative downtime distribution. Let

$$\begin{aligned} D_t &= \text{the cumulative downtime in time period } t \\ &= t - h_t \end{aligned} \quad (17)$$

Hence, Eq. 16 can be rewritten as

$$P((t - D_t) < s) = \sum_{n=0}^{\infty} [G_n(t-s) - G_{n+1}(t-s)] F_{n+1}(s) \quad , \quad (18)$$

or

$$P(D_t > (t-s)) = \sum_{n=0}^{\infty} [G_n(t-s) - G_{n+1}(t-s)]F_{n+1}(s) \quad . \quad (19)$$

Substituting

$$d = t - s \quad , \quad (20)$$

one finally has

$$P(D_t > d) = \sum_{n=0}^{\infty} [G_n(d) - G_{n+1}(d)]F_{n+1}(t-d) \quad . \quad (21)$$

Using standard renewal relations, Eq. 21 can also be written in terms of the distribution of the number of downtime and uptimes. Let

$P[N_r(d) = n]$  = the probability that the number of repairs or restorations  $N_r(d)$  in period  $d$  equals  $n$ , and

$P[N_f(t-d) > n]$  = the probability that the number of failures  $N_f(t-d)$  in period  $t-d$  is greater than  $n$ .

Then

$$P(D_t > d) = \sum_{n=0}^{\infty} P(N_r(d) = n) P(N_f(t-d) > n) \quad . \quad (22)$$

To determine  $P(D_t > d)$  either Eq. 21 or Eq. 22 can be used.

Equation 21 is generally more useful when the downtime and uptime distributions are the prime interest and not the number of failures or repairs occurring. Since the successive terms in the summation in Eqs. 21 and 22 generally decrease significantly with increasing  $n$ , only a relatively few terms need be actually calculated. The error in truncating the series can be bounded using standard approaches to determine the precision of the truncated series.

To determine  $P(D_t > d)$  using either Eq. 21 or 22, the uptime distribution  $F(x)$  and the downtime distribution  $G(x)$  are required as input. From these distributions, the convolutions  $G_n(x)$  and  $F_n(x)$  are calculated and are used in the equations.

The uptime and downtime distributions are obtained from historical data and from knowledge of a component's characteristics. Parametric distributions such as the gamma and normal can be used where justified. In many cases, only partial information will be known or will be meaningfully obtainable from data. For example, only the approximate median and spread may be obtainable without assuming unjustifiable distribution shapes. In these partial information cases, bounds can be obtained on  $G(x)$  and  $F(x)$  and hence on  $P(D_t > d)$ . Sensitivity studies can also be performed using various parametric families for  $G(x)$  and  $F(x)$  which are constrained to be consistent with the available partial information.

The calculation of the convolutions  $G_n(x)$  and  $F_n(x)$  can be explicitly done in some cases. For example, convolutions of normal distributions and gamma distributions are other normal and gamma distributions, respectively. Convolutions of mixtures of normals and convolutions of mixtures of gammas are expressible as finite sums of normals and gammas, respectively. These mixture models can be particularly appropriate when there are distinct failure modes requiring significantly different repair times. When the convolutions are not explicitly determinable, then discrete numerical evaluations can be straightforwardly performed. Since the convolution integrals are one dimensional, standard, efficient techniques can be used.

In addition to numerical evaluation of convolution integrals, approximate solutions can be obtained through various techniques. These methods of solution will be discussed in Chapter 4.

Finally, the alternating renewal process can be applied to evaluate the cumulative downtime distribution of a system composed of several components. In this case, the overall cumulative downtime of the system can be expressed as:

$$d = \sum_{i=1}^n d_i \quad , \quad (23)$$

where,  $n$  is the number of components within the system and,  $d_i$  is the individual downtime of each component.

Since  $d_i$  are the random variables sampled from the distribution of cumulative downtime of each individual component, the distribution of  $d$  can be obtained through the convolutions of individual downtime distributions. Again, methods such as stratified sampling can be used to facilitate the evaluation of convolution integrals. This will be discussed in Chapter 4.

### 3.3 Marked Markov Process

The other concept for estimating the cumulative downtime distribution is the use of general Markov processes. This concept was investigated as part of this study. The cumulative downtime in terms of Markov terminology is considered to be the cumulative residence time in the repair (down) state. Therefore, it is needed to set up the Markov equations with explicit consideration of the cumulative residence time of the down state (e.g., state  $j$ ),  $\tau_j$ . The set of equations derived in

this manner is called here marked Markov process. These equations are in integral form with extensive involvement of delay operators (representative of memory) which are not generally amenable to analytical closed-form solutions.

To derive the equations for marked Markov processes, we initially need to define some terms. These are:

$P_{i\ell}(\tau, \tau_j)$ : The pdf for a transition from state  $i$  to state  $\ell$  ( $\ell \neq i$ ) with the residence time of  $\tau$  in state  $i$  and cumulative residence time  $\tau_j$  in state  $j$ . From here on  $\tau_j$  will be referred to as "memory time",

$\eta_i(t, \tau_j)$ : The entrance probability to state  $i$  which is the probability that the system enters state  $i$  at some time  $t$  with the memory time  $\tau_j$ ,

$e_i(t, \tau_j)$ : The exit probability of state  $i$  which is similar to the entrance probability and is defined as the probability that the system leaves state  $i$  at some time  $t$  with the memory time  $\tau_j$ , and

$\phi_i(t, \tau_j)$ : The residence probability of state  $i$  which is the probability that the system resides in state  $i$  at some time  $t$  with the memory time  $\tau_j$ .

Finally, we define  $h_i(\tau, \tau_j)$  as the overall transition probability out of state  $i$ , that is:

$$h_i(\tau, \tau_j) = \sum_{\ell \neq i} P_{i\ell}(\tau, \tau_j) \quad . \quad (24)$$

The governing equations for a marked Markov process can be set up using exhaustive reasoning; for example, the probability of entering state  $j$  at time  $t$  with memory time  $\tau_j$  results from a process which entered a state  $i$  at sometime  $\tau'$  less than  $t$ , with a memory time  $\tau_j$ ; resided in state  $i$  for a period  $t-\tau'$  and finally made transition to state  $j$  at time  $t$  (note memory time  $\tau_j$  would stay constant during this process since  $\tau_j$  varies only with the residence time of state  $j$ ). Therefore, to calculate the entrance probability of state  $j$ , the above process has to be summed over all possible values of  $i$  and integrated over all values of  $\tau'$  less than  $t$  and greater than  $\tau_j$ , that is:

$$\eta_j(t, \tau_j) = \sum_{i=1}^n \int_{\tau'=\tau_j}^t \eta_i(\tau', \tau_j) P_{ij}(t-\tau', \tau_j) d\tau' \quad . \quad (25)$$

Following the same type of reasoning and the definitions given early, a set of equations describing a marked Markov process can be established. These equations are listed in Table 2 (Eqs. 26-33).

The integral equations can be solved by computer using numerical schemes. For the purpose of this dissertation, the analytical solutions for a simple case appear to be more useful than the numerical solutions for gaining insights regarding the distribution of  $\tau_j$ . One approach for dealing with the analytical solutions is through the principle of moment propagation. This is the approach which will be discussed in detail in the remainder of this chapter.

Let us define the expectation of  $\tau_j^k$  with respect to any function  $w(t, \tau_j)$  under the restriction of  $\tau_j \leq t$  as follows:

Table 2. The Governing Equations for Marked Markov Process

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$$\eta_j(t, \tau_j) = \sum_{i=1}^N \int_{\tau'=\tau_j}^t \eta_i(\tau', \tau_j) P_{ij}(t-\tau', \tau_j) d\tau' \quad (26)$$

$$e_j(t, \tau_j) = \int_{\theta=0}^{\tau_j} f_j(t-\theta, \tau_j - \theta) h_j(\theta, \tau_j) d\theta \quad (27)$$

FOR  $l \neq j$

$$\eta_l(t, \tau_j) = \sum_{\substack{i=1 \\ i \neq j}}^N \int_{\tau'=\tau_j}^t \eta_i(\tau', \tau_j) P_{il}(t-\tau', \tau_j) d\tau' \\ + \int_{\theta=0}^{\tau_j} f_j(t-\theta, \tau_j - \theta) P_{jl}(\theta, \tau_j) d\theta \quad (28)$$

$$e_l(t, \tau_j) = \int_{\tau'=\tau_j}^t \eta_l(\tau', \tau_j) h_l(t-\tau', \tau_j) d\tau' \quad (29)$$

$$f_j(t, \tau_j) = \int_{\theta=0}^{\tau_j} f_j(t-\theta, \tau_j - \theta) [1 - h_j(\theta, \tau_j)] d\theta \quad (30)$$

$$\phi_l(t, \tau_j) = \int_{\theta=0}^{t-\tau_j} \eta_l(t-\theta, \tau_j) [1 - h_l(\theta, \tau_j)] d\theta \\ = \int_{\tau'=\tau_j}^t \eta_l(\tau', \tau_j) [1 - h_l(t-\tau', \tau_j)] d\tau' \quad (31)$$

$$\sum_{i=1}^N \phi_i(t, \tau_j) = \Psi(t, \tau_j) \quad (32)$$

$$\int_0^t \Psi(t, \tau_j) d\tau_j = 1 \quad (33)$$


---

$$w_k(t) = E (\tau_j^k \text{ w.r.t } w(t, \tau_j)) = \int_0^t \tau_j^k w(t, \tau_j) d\tau_j \quad . \quad (34)$$

Therefore, in terms of the variables of the marked Markov process we can define:

$$\eta_{k,\ell}(t_j) = E (\tau_j^k \text{ w.r.t } \eta_\ell(t, \tau_j)) = \int_0^t \tau_j^k \eta_\ell(t, \tau_j) d\tau_j \quad , \quad (35)$$

$$\phi_{k,\ell}(t_j) = E (\tau_j^k \text{ w.r.t } \phi_\ell(t, \tau_j)) = \int_0^t \tau_j^k \phi_\ell(t, \tau_j) d\tau_j \quad . \quad (36)$$

The new form of the equations of the marked Markov process then is to be obtained under the above transformations. Let us assume for now that the transition probabilities are not a function of  $\tau_j$ , that is:

$$P_{ik}(t, \tau_j) = P_{ik}(t) \quad . \quad (37)$$

Now, let us take Eq. 26 and multiply both sides of the equation by  $\tau_j^k$  and integrate over all values of  $\tau_j$  belong to an interval from 0 to  $t$ ,  $\tau_j \in (0, t)$ , that is:

$$\eta_{j,k}(t) = \sum_{i=1}^n \int_{\tau_j=0}^t \int_{\tau'=0}^{\tau_j} \tau_j^k \eta_i(\tau', \tau_j) P_{ij}(t-\tau') d\tau' d\tau_j \quad . \quad (38)$$

If we look at the region of the integration in Eq. 38 as shown in Figure 1 by a vertical strip for a given  $\tau_j$ , the value of the integral would not change if the integration is performed in horizontal strips for a given  $\tau'$ , that is:

$$\eta_{j,k}(t) = \sum_{i=1}^n \int_{\tau'=0}^t \int_{\tau_j=0}^{\tau} \tau_j^k \eta_i(\tau', \tau_j) P_{ij}(t-\tau') d\tau' d\tau_j \quad . \quad (39)$$

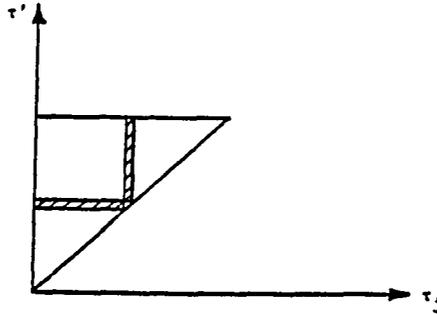


Figure 1. Alternative regions of integration for Eq. 38.

Now, if we integrate first with respect to  $\tau_j$ , the equation can be written as follows:

$$\eta_{j,k}(t) = \sum_{i=1}^n \int_{\tau'=0}^t \eta_{i,k}(\tau') P_{ij}(t-\tau') d\tau' d\tau_j \quad . \quad (40)$$

Now, let us apply a similar approach to Eq. 27, that is:

$$e_{j,k}(t) = \int_{\theta=0}^{\tau_j} \int_{\tau_j=0}^t \tau_j^k \eta_j(t-\theta, \tau_j-\theta) h_j(\theta) d\theta \quad , \quad (41)$$

using the alternate region of integration (horizontal strip rather than vertical strip) in Figure 2, Eq. 41 becomes:

$$e_{j,k}(t) = \int_{\theta=0}^t \int_{\tau_j=\theta}^t \tau_j^k \eta_j(t-\theta) h_j(\theta) d\theta \quad . \quad (42)$$

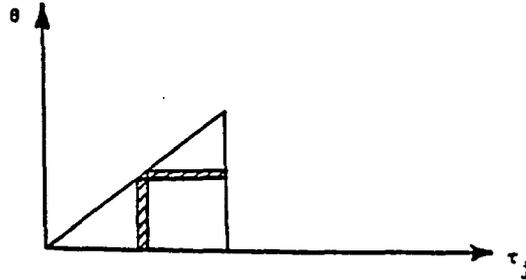


Figure 2. Alternative regions of integration for Eq. 41

Let  $\tau_j' = \tau_j - \theta$ ;

$$e_{j,k}(t) = \int_{\theta=0}^t \int_{\tau_j'=0}^{t-\theta} (\tau_j'+\theta)^k \eta_j(t-\theta, \tau_j') h_j(\theta) d\theta \quad , \quad (43)$$

expand  $(\tau_j' + \theta)^k$  and substitute for the expectations;

$$e_{j,k}(t) = \sum_{m=0}^k \int_{\theta=0}^t \left[ \binom{k}{m} \theta^{k-m} h_j(\theta) \right] \eta_{j,m}(t-\theta) d\theta \quad . \quad (44)$$

The remainder of the equations of a marked Markov process are in the same form as Eqs. 26 and 27; therefore, their modified forms under the moment transformation equations of the marked Markov process for entrance and residence probabilities are shown in Table 3 (Eqs. 46-49). The Laplace transforms of these equations are given in Table 4 (Eqs. 50-54).

After this mathematical exercise, it is important to note some important results that we have already obtained. These are:

Table 3. Marked Markov Equations Under Moment Transformation

$$f_{j,k}(t) = \sum_{i=1}^n \int_{\tau'=0}^t f_{i,k}(\tau') P_{ij}(t-\tau') d\tau' \quad (45)$$

$$\eta_{\ell,k}(t) = \sum_{i=1}^n \int_{\tau'=0}^t \eta_{i,k}(\tau') P_{i\ell}(t-\tau') d\tau' + \sum_{m=0}^k \binom{k}{m} \int_{\theta=0}^t p_{j\ell}(\theta) \cdot \theta^{k-m} \quad (46)$$

$$\phi_{j,k}(t) = \sum_{m=0}^k \binom{k}{m} \int_{\theta=0}^t \theta^{k-m} (1-h_j(\theta)) \eta_{j,m}(t-\theta) d\theta \quad (47)$$

$$\phi_{\ell,k}(t) = \int_{\tau'=0}^t \eta_{\ell,k}(\tau') (1-h_{\ell}(t-\tau')) d\tau' \quad (48)$$

$$M_k = \sum_{i=1}^n \phi_{i,k}(t) = \sum_{\substack{\ell=1 \\ \ell \neq j}}^n \int_{\tau'=0}^t \eta_{\ell,k}(\tau') (1-h_{\ell}(t-\tau')) d\tau' + \sum_{m=0}^k \binom{k}{m} \int_{\theta=0}^t \theta^{k-m} (1-h_j(\theta)) \eta_{j,m}(t-\theta) d\theta \quad (49)$$

where  $M_k$  is the unconditional moments of  $\tau_j$  and  $M_0 = 1$ .

Table 4. Laplace Transform of Marked Markov Equations  
Under Moment Transformation

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$$\eta_{j,k}(s) = \sum_{i=1}^n \eta_{i,k}(s) P_{ij}(s) \quad (50)$$

$$\eta_{\ell,k}(s) = \sum_{\substack{i=1 \\ i \neq j}}^n \eta_{i,k}(s) P_{i\ell}(s) \sum_{m=0}^k \binom{k}{m} \eta_{j,m}(s) P_{j\ell}^{k-m}(s) \quad (51)$$

where

$P_{j\ell}^{k-m}(s) = \text{Lap}[\theta^{k-m} P_{j\ell}(\theta)]$  and  $f$  stands for Laplace transformation.

$$\phi_{j,k}(s) = \sum_{m=0}^k \binom{k}{m} \eta_{j,m}(s) \xi_j^{k-m}(s) \quad (52)$$

where

$$\xi_j^{k-m}(s) = [\theta^{k-m}(1-h_j(\theta))]$$

$$\phi_{\ell,k}(s) = \eta_{\ell,k}(s) \xi_{\ell}(s) \quad (53)$$

where

$$\xi_{\ell}(s) = \xi_{\ell}^0(s) = [1-h_{\ell}(\theta)]$$

$$M_k(s) = \sum_{\substack{\ell=1 \\ \ell \neq j}}^n \eta_{\ell,k}(s) + \sum_{m=0}^k \binom{k}{m} \xi_j^{k-m}(s) \eta_{j,m}(s) \quad (54)$$


---

1. For  $k=0$ , the marked Markov equations are transformed to semi-Markov processes for non-exponential transition probabilities and to Markov processes for exponential transition probabilities. This can be easily shown from the equations given in Table 3 which will yield the integral Markov equations when  $k$  is set to zero.
2. Knowing the solutions of a semi-Markov or Markov process (whichever is applicable), the solutions of a marked Markov process can be obtained. To show this, let us look at Eqs. 50 and 51 in Table 4. The matrix representation of these equations is:

$$\bar{\eta}_k(s) = \bar{P}(s) \eta_k(s) + \bar{U}_{k-1}(s) \quad , \quad (55)$$

where the elements of each vector and the matrix are:

$$\eta_k^i(s) = \eta_{k,i}(s) \quad , \quad (56)$$

$$P^{i,l}(s) = P_{i,l}(s) \quad , \quad (57)$$

$$U_{k-1}^l(s) = \sum_{m=0}^{k-1} \binom{k}{m} P_{j,l}^{k-m}(s) \eta_{j,m}(s) \quad . \quad (58)$$

Since Eq. 55 is in a recursive form, knowing  $\eta_{j,0}(s)$  which is the solution of a semi-Markov process, would determine  $\underline{U}_0(s)$ . Equation 55 then can be used to solve for  $\eta_1(s)$  which in return allows us to determine  $U_1(s)$ .  $U_1(s)$  can then be evaluated. Then,  $U_1(s)$  can be determined from the  $j$ -th element of  $h(s)$

which can be used to evaluate  $\eta_2(s)$ . This recursive nature of the equations would allow the determination of  $\eta_k(s)$  for every value of  $k$ , only if the  $\eta_{j,0}(s)$  is known. This recursive form can be written simply as:

$$\bar{\eta}_k(s) = (I - \bar{P}(s))^{-1} \bar{U}_{k-1}(s) \quad . \quad (59)$$

3. If  $S=S_0$  is a pole of the semi-Markov or Markov process, that is  $\det (I-P(s_0)) = 0$  , (60)

then  $\eta_{k,1}(s)$  will have a  $(k+1)$  repeated pole of  $S_0$ . This is a straightforward conclusion from the form of the equations. If  $S_0$  is a pole of  $(I-P(s))$ , then  $S_0$  is a pole of  $\eta_{j,0}(s)$  and since  $S_0$  cannot be a pole of  $P(s)$  because it is pole of  $(I-P(s))$ , then  $S_0$  would be a pole for  $U_0(s)$ .

The importance of this Lemma is obvious when one deals with non-absorbing stable Markov process: the residence probabilities will approach a constant as time approaches infinity. In this case,  $S_0 = 0$  is a pole of a semi-Markov/Markov process. Therefore, it can be concluded that the  $k$ th moments of  $\tau_j$  from a marked Markov process will approach  $\alpha_k t^k$  as time approaches infinity (has a dominant pole of  $S = 0$  with  $k+1$  repetitions).

4. The observations that have been made so far are all under the assumption that the transition probabilities are not functions of memory time  $\tau_j$ . This assumption was made to facilitate the derivation of the equations for transformed marked Markov proc-

ess. In many practical applications in reliability areas as well as in physical modellings, this assumption may not be valid. Even in the case of this dissertation, namely the determination of allowable cumulative outage time (ACOT), one can hypothesize that as the accumulation time in the down state increases, the plant staff would try to minimize the downtime of the component whenever it fails. The effect of such a strategy not only results in dependency of transition parameters of down state upon the memory time " $\tau_j$ ", but also may imply that the transition parameters of other states are functions of memory time as well.

To address such a problem it is important to note that the memory time  $\tau_j$  is considered as a random variable throughout this modeling effort. When dealing with functions of random variables in the context of differential or integral equations, it is usually not fruitful to account for the random variable in an explicit manner as is usually done for deterministic variables.<sup>1</sup> To illustrate this point, let us take Eq. 39 and assume that the transition parameter  $P_{ij}$  is a function of both  $t-\tau'$  and  $\tau_j$ . If we write  $P_{ij}(t-\tau', \tau_j)$  in a Taylor series of  $\tau_j$  and carry out the integration and Laplace transform to get an equivalent equation for Eq. 50, one would obtain:

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<sup>1</sup>This is one reason why, when dealing with the random variable through differential/integral equations, the use of either Ito or Stratonovich calculus is recommended.

$$\eta_{j,k}(s) = \sum_{q=0}^{\infty} \sum_{i=1}^n \eta_{i,k+q}(s) P_{ij}(s) \quad . \quad (61)$$

As the form of Eq. 61 suggests, the recursive behavior has been diminished. Therefore, the analytical solutions using this approach cannot be obtained systematically. However, the original equation can still be solved using numerical analysis.

We may also follow a slightly different philosophy. Rather than explicit accounting of  $\tau_j$ , we might be able to find a systematic iterative process for obtaining the solutions. In this algorithmic approach, we assume that the transition parameters are not an explicit function of  $\tau_j$ , rather they are functions of the expectation of  $\tau_j$  (e.g., the first moment). Now one must determine which expectation of  $\tau_j$  is to be used: the conditional or unconditional expectation. In terms of the marked Markov process and its application to allowable cumulative downtime, the proper expectation to be used is the expectation of  $\tau_j$  conditional, or at the time of entering a given state. If this approach is taken, the solutions can be obtained through a systematic iterative process. This iterative process is described in the following three steps:

1. Guess the functional form for the expectations of  $\tau_j$  as a function of  $t$ , e.g., for state  $i$  the functional form of the  $\eta_{i,1}^{(0)}(t)/\eta_{i,0}^{(0)}(t)$  is to be guessed where  $(0)$  indicates that this is the function for zero iteration.
2. Now define the transition parameters of state  $i$  as a function of  $t$ , that is:

$$P_{ik}(t, \tau_j) = P_{ik}\left(t, \frac{n_{i,1}^{(0)}(t)}{n_{1,0}^{(0)}(t)}\right) P_{ij}^{(0)}(t) \quad (62)$$

3. Solve the equations to get  $\eta_{i,1}^{(1)}(t)$  and perform the second iteration. Continue iterations until some preset convergence criteria is met.

To assure the best convergence of this process, one should start with a proper form of  $\eta_{i,1}^{(0)}(t)/\eta_{1,0}^{(0)}(t)$ . This can be done by noting that  $\tau_j$  is a monotonically increasing function of  $t$ . For stable and non-absorbing Markov process, it asymptotically increases as a linear function of  $t$ . The proportionality factor can be determined from the asymptotic solutions of the underlying Markov process when both  $t$  and  $\tau_j$  approach infinity.

This methodology of the marked Markov process, although developed here, has not been computerized to facilitate applications. The solution schemes for these equations are complex and require a comparatively large effort; therefore, it is left for future work in this area. However, to illustrate the usefulness of the methodologies, the analytical solutions for a specific case are derived here.

Consider a process consisting of two states, namely states 1 and 2. The interest is to estimate the cumulative residence time of state 2 at some time  $t$  under the assumption that the transition parameters are not functions of the memory time  $\tau_2$  and the system entered state 1 at time zero with zero memory time.

In this case, Eq. 59 can be written in the following form:

$$\begin{vmatrix} \eta_{1,k}(s) \\ \eta_{2,k}(s) \end{vmatrix} = \frac{1}{1-P_{12}^0(s) P_{21}^0(s)} \begin{vmatrix} 1 & P_{21}^0(s) \\ P_{12}^0(s) & 1 \end{vmatrix} \begin{vmatrix} U_{k-1}(s) \\ 0 \end{vmatrix}, \quad (63)$$

where

$$U_{k-1}(s) = \begin{cases} 1 & \text{for } K = 0: \text{ Laplace transform of} \\ & \text{delta function rep-} \\ & \text{resenting the entrance} \\ & \text{to State 1 at } t=0. \\ \sum_{m=0}^{k-1} \binom{k}{m} \eta_{2,m}(s) P_{21}^{k-m}(s) & \text{for } K = 1, 2, \dots \end{cases}$$

The general expressions for  $K = 0, 1,$  and  $2$  are

$$\eta_{1,0}(s) = \frac{1}{1-P_{12}^0(s) P_{21}^0(s)}, \quad (64)$$

$$\eta_{1,1}(s) = \frac{P_{12}^0(s) P_{21}^1(s)}{(1-P_{12}^0(s) P_{21}^0(s))^2}, \quad (65)$$

$$\begin{aligned} \eta_{1,2}(s) = & \frac{1}{(1-P_{12}^0(s) P_{21}^0(s))^3} P_{12}^0(s) P_{21}^2(s) (1-P_{12}^0(s) P_{21}^0(s)) \\ & + 2(P_{12}^0(s) P_{21}^1(s))^2, \end{aligned} \quad (66)$$

$$\eta_{2,k}(s) = \eta_{1,k}(s) P_{12}^0(s). \quad (67)$$

Now if we consider the case where transition probabilities are exponential with a constant rate of  $\lambda$  and  $\mu$  for states 1 and 2, then:

$$P_{12}^0(s) = \lambda/(s+\lambda), \quad (68)$$

$$P_{21}^0(s) = \mu/(s+\mu), \quad (69)$$

$$P_{21}^1(s) = \frac{\mu}{(s+\mu)^2}, \quad (70)$$

$$p_{21}^2(s) = \frac{2\mu}{(s + \mu)^3}, \quad (71)$$

Therefore,

$$\eta_{1,0}(s) = \frac{(s + \lambda)(s + \mu)}{s(s + (\lambda + \mu))}, \quad (72)$$

$$\eta_{1,1}(s) = \frac{\mu\lambda(s + \lambda)}{s^2(s + (\lambda + \mu))^2}, \quad (73)$$

$$\eta_{1,2}(s) = \frac{2\mu\lambda(s + \lambda)^2}{s^3(s + (\lambda + \mu))^3}. \quad (74)$$

Applying the residue theorem to the above equations (i.e., Eqs. 72-74) the inverse Laplace transforms can be obtained. These are given in Eqs. 75-77.

$$\bar{p}_{1,0} = \delta(t) + \mu\lambda/(\mu + \lambda) (1 - e^{-(\mu + \lambda)t}), \quad (75)$$

$$\begin{aligned} \eta_{1,1}(t) &= \frac{\mu\lambda(\mu - \lambda)}{(\mu + \lambda)^3} (1 - e^{-(\mu + \lambda)t}) \\ &\quad + \frac{\mu\lambda^2}{(\mu + \lambda)^2} t (1 - e^{-(\mu + \lambda)t}) \end{aligned} \quad (76)$$

$$\begin{aligned} \eta_{1,2}(t) &= 2\mu\lambda(\mu^2 - 4\mu\lambda + \lambda^2)/(\mu + \lambda)^5 (1 - e^{-(\mu + \lambda)t}) \\ &\quad + (2\mu\lambda^2(2\mu - \lambda)t)/(\mu + \lambda)^4 (1 + e^{-(\mu + \lambda)t}) \\ &\quad + (2\lambda\mu^3 t^2)/(\mu + \lambda)^3 (1 - e^{-(\mu + \lambda)t}). \end{aligned} \quad (77)$$

The moments of the distribution of the cumulative outage time can be calculated using Eq. 54 for  $M_k(s)$ . In addition to unconditional moments of the distribution of cumulative outage time, conditional moments can also be calculated. For example:

$$m_{1,1} = \left\{ \begin{array}{l} \text{The expectation of } \tau_j \text{ given that} \\ \text{the system enters state 1 at time } t \end{array} \right\} = \eta_{1,1}(t) / \eta_{1,0}(t) \quad (78)$$

or, in general,

$$m_{1,k} = \left\{ \begin{array}{l} \text{The } k\text{th moments of } \tau_j \text{ given that} \\ \text{the system enters state 1 at time } t \end{array} \right\} = \eta_{1,k}(t) / \eta_{1,0}(t) \quad (79)$$

The asymptotic behaviors of these conditional expectations for large values of  $t$  are important and can be directly shown to be:

$$\lim_{t \rightarrow \infty} m_{1,1}(t) = \lambda t / (\mu + \lambda) \quad , \quad (80)$$

$$\lim_{t \rightarrow \infty} m_{1,2}(t) = 2\mu^2 t^2 / (\mu + \lambda)^2 \quad . \quad (81)$$

Therefore, the conditional mean and variance of  $\tau_j$  for the above case, namely  $m_1$  and  $\sigma_1$ , as  $t$  approaches infinity are:

$$m_1 = (\lambda t) / (\mu + \lambda) \quad , \quad (82)$$

$$\sigma_1 = t(2\mu^2 - \lambda^2)^{0.5} / (\mu + \lambda) \quad . \quad (83)$$

It is interesting to note that the mean calculated in this manner has a close relationship to the mean calculated by the compound Poisson process when the operating period is reduced (corrected) by the expectation of cumulative downtime period, that is

$$\lambda(t-d) = \mu d \quad , \quad (84)$$

or

$$d = \lambda t / (\mu + \lambda) \quad , \quad (85)$$

where  $d$  is the expectation of the cumulative outage time. Unfortunately, for the higher moments, a compound Poisson process yields a poor approximation to the exact results.

#### 4. SOLUTION TECHNIQUES

In this chapter, various solution techniques for evaluating the cumulative downtime distribution are discussed. This chapter consists of four sections each describing a specific method for determination of cumulative downtime distribution. These are:

- 1) Solutions when the distribution of repair time and downtime frequency have compact convolution functions.
- 2) Solutions when the distribution of downtime frequency is a Poisson distribution and the distribution of repair time has a general form (not necessarily having compact convolution functions) with the first several moments in existence (compound Poisson process).
- 3) Solutions when the distribution of repair time and downtime frequency have general forms with the first several moments in existence.
- 4) Solutions when the distribution of both repair time and downtime frequency have general forms with no restriction on the existence of moments.

##### 4.1 Solutions For The Distribution With Compact Convolutions

As discussed earlier in Chapter 3 (Eq. 21), the exceedance probability for the cumulative downtime distribution can be evaluated by:

$$P(Dt > d) = \sum_{n=0}^{\infty} [G_n(d) - G_{n+1}(d)] F_{n+1}(t-d) \quad , \quad (86)$$

where:

Dt: Cumulative downtime in period t,

d: Allowable cumulative downtime for period t,

$G_n(d)$ : The probability that the sum of n downtime durations is less than or equal to d,

$F_{n+1}(t-d)$ : The probability that sum of n+1 uptime durations is less than or equal to (t-d), and

$$G_0(x) = F_0(x) = 1 \quad , \quad (87)$$

$$G_n(x) = \int_{-\infty}^x G_{n-1}(x-x')g(x')dx' = \int_{-\infty}^x G_n(x-x') dG(x') \quad , \quad (88)$$

$$F_n(x) = \int_{-\infty}^x F_{n-1}(x-x')f(x')dx' = \int_{-\infty}^x F_{n-1}(x-x') dF(x') \quad , \quad (89)$$

Here  $f(x)$  and  $g(x)$  are the probability distribution functions (pdf) for the downtime occurrence probability and the restoration duration probability, respectively.

There are several known distributions in the areas of probabilistic risk analyses where the form of  $G_n(x)$  and  $F_n(x)$  can be simply determined. These distributions all share a common characteristic, namely the existence of known analytical compact forms for their convolutions. Therefore,  $G_n(x)$  and  $F_n(x)$  can be determined through a simple integration routine. Some of these distributions are: normal, exponential, gamma, Cauchy, one sided stable, etc. A variety of other distributions can be generated from these distributions through a linear mixture equation such as:

$$f_m(x) = p_1 \cdot f_1(x) + (1-p_1)f_2(x) \quad (90)$$

where  $p_1$  and  $(1-p_1)$  are the mixture probabilities, and  $f_1(x)$  and  $f_2(x)$  are two distributions from the set of distributions with compact convolution forms.

The convolution functions of the generated mixture distribution when both  $f_1(x)$  and  $f_2(x)$  belong to the same family (e.g., both are exponentials but with different parameters) can be evaluated through Eq. 91.

$$f_m(x) = \sum_{j=0}^k \binom{k}{j} p_1^j (1-p_1)^{k-j} f_1^j(x) * f_2^{k-j}(x) \quad , \quad (91)$$

where  $f_1^j(x)$  and  $f_2^{k-j}(x)$  stand for  $j$ th and  $(k-j)$ th convolutions of  $f_1$  and  $f_2$ , respectively, and "\*" stands for convolution operator. When  $f_1(x)$  and  $f_2(x)$  are from the same family of distributions then the convolution of  $f_1^j(x)$  and  $f_2^{k-j}(x)$  will have compact form.

A set of commonly used distributions with the above characteristic have been modeled in the computer program developed for this study. A list of these distributions and their functional forms is provided in Table 5.

#### 4.2. Solution Technique for Compound Poisson Process

As discussed earlier (Chapter 3 (Eq. 10)), the characteristic function for the cumulative downtime distribution function using the compound Poisson process can be obtained by:

$$F(\theta) = e^{-\lambda t(1-G(\theta))} \quad , \quad (92)$$

where  $F(\theta)$  and  $G(\theta)$  are the characteristic functions for cumulative

Table 5. Options Modeled for Distributions with Known Compact Convolution Functional Forms

		Variables
Normal	$\eta(x   \mu, \sigma) = \frac{1}{\sqrt{2\pi} \sigma} e^{-1/2(x-\mu)/\sigma^2}$	$\mu, \sigma$
Exponential	$f(x   \lambda, t_0) = \lambda e^{-\lambda(t-t_0)}$	$t_0, \lambda$
Gamma	$f_{\alpha, \gamma}(x) = \frac{1}{\Gamma(\gamma)} \alpha^\gamma x^{\gamma-1} e^{-\alpha x}$	$\gamma, \alpha$
Cauchy	$j_t(x) = \frac{1}{\pi} \cdot \frac{1}{t^2+x^2}$	$t$
One-Sided Stable	$f_\alpha(x) = \frac{\alpha}{\sqrt{2\pi}} \cdot \frac{1}{\sqrt{x^3}} e^{-1/2 \alpha^2/x}$	$\alpha$
Mixture of two exponentials	$f_{me}(x) = P_1 f(x   \lambda_1, t_1) + P_2 f(x   \lambda_2, t_2)$	$P_1, \lambda_1, t_1$ $P_2, \lambda_2, t_2$
Mixture of two normals	$f_{mn}(x) = P_1 \eta(x   \mu_1, \sigma_1) + P_2 \eta(x   \mu_2, \sigma_2)$	$P_1, \mu_1, \sigma_1$ $P_2, \mu_2, \sigma_2$
Mixture of two gammas	$f_{mg}(x) = P_1 f_{\alpha_1, \gamma_1}(x) + P_2 f_{\alpha_2, \gamma_2}(x)$	$P_1, \alpha_1, \gamma_1$ $P_2, \alpha_2, \gamma_2$

downtime and single downtime distribution functions respectively and  $\lambda$  is the uptime to downtime transition rate.

Generating the moments of the cumulative downtime distribution function according to Eq. 9, one obtains the following relationship:

$$M_f^{k+1} = (\lambda t) \left| \sum_{j=1}^k \binom{k}{j} M_f^j \cdot M_g^{k-j} + M_g^{k+1} \right| \quad (93)$$

where  $M_f$  and  $M_g$  are the  $j$ th moment of cumulative downtime distribution and the  $k$ th moment of the single downtime distribution, respectively.

Therefore, the moments of cumulative downtime distribution can easily be calculated from the moments of the single downtime distribution. Furthermore, the number of moments are preserved (e.g., if seven moments are available for the distribution of single downtime duration, this technique would provide seven moments for the cumulative downtime distribution).

The general question that still remains is how to estimate a distribution function when only a limited number of moments are known. A unique technique was used in this study which will be described in this section.

In mathematical language, the problem of interest is to determine  $f(\theta)$  such that the following relation is satisfied:

$$M_k = \int_0^{\infty} \theta^k f(\theta) d\theta \quad k = 0, 1, 2, \dots, n \quad (94)$$

where  $M_0=1$  and  $M_k$  is the  $k$ th moment of  $f(\theta)$ .

One approach which is commonly used is to develop the functional form of a bounding distribution (a distribution with maximum uncertainty). These types of bounding distributions are known as maximum entropy distributions. The maximum entropy distribution is the distribution which maximize the entropy function. The entropy function itself is a measure of uncertainty (it may also be looked at as a measure of information content in a distribution) and if it is maximized would provide a functional form for the distribution which maximize the uncertainty and minimize the information<sup>22</sup> (that is specifying the functional form would add minimum information to the actual information).

The entropy function is defined by:

$$S = - \int f(\theta) \cdot \ln(f(\theta)) \cdot d\theta \quad . \quad (95)$$

Therefore, for the problem defined earlier "S" has to be maximized under the constraints imposed by Eq. 94. Using the Lagrangean multipliers, the following function is to be maximized:

$$\psi = S - \sum_{i=0}^N \lambda_i \cdot M_i \quad . \quad (96)$$

Therefore,

$$\frac{\partial \psi}{\partial f} = - [1 + \ln(f(\theta)) + \sum_{i=0}^N \lambda_i \cdot \theta^i] d\theta = 0 \quad . \quad (97)$$

This yields,

$$f(\theta) = e^{-\sum_{i=0}^N \alpha_i \theta^i} \quad , \quad (98)$$

where

$$\alpha_i = \begin{cases} -(1 + \lambda_0) & \text{for } i=0 \\ -\lambda_i & \text{for } i \neq 0 \end{cases} \quad (99)$$

Knowing the functional form of  $f(\theta)$  and the  $k+1$  restrictions imposed by Eq. 94,  $k+1$  values for  $\lambda_i$ 's can conceptually be determined. However, this is not a simple task to undertake. The functional form given by Eq. 98 does not have analytical integrals. Therefore, for the cases when more than three moments are known, an iterative computer program must be written to solve simultaneously for the values of  $\lambda_i$ 's. Past experience has shown<sup>23</sup> that this type of solution technique has very poor convergence when the number of moments exceeds five or six. An alternate method is discussed in Reference 23, using an adaptive process which assumes that the argument of the exponential (i.e.,  $\sum \alpha_i \cdot \theta^i$ ) in Eq. 98 is a truncated Taylor expansion of a function from a set of predetermined functions. The best solution is then obtained by applying each of these predefined functions and investigating which function can best satisfy the restrictions imposed by the information available. Although this approach has provided some good results in the past, it is strongly dependent on the selection of the predefined functions and generally may not provide good results.

To discuss the approach used in this study, another technique is to be first discussed. Referring back to our original problem, we need to determine a function  $f(\theta)$  such that Eq. 94 is to be satisfied.

Let  $w(\theta)$  be defined as :

$$w(\theta) = e^{-\theta} f(\theta) \quad . \quad (100)$$

Furthermore, let us assume that "Z" defined by Eq. 101 exists:

$$Z = \int_0^{\infty} e^{-\theta} w^2(\theta) d\theta = \int_0^{\infty} e^{-\theta} f^2(\theta) d\theta \quad , \quad (101)$$

This Z exists only if  $f(\theta)$  approaches zero for large values of  $\theta$  faster than  $e^{-.5\theta}$ . The condition on  $f(\theta)$  can be further weakened if one performs a change of variable from  $\theta$  to  $\alpha\theta$ . In this case the existence of Z is assumed as long as  $f(\theta)$  approaches zero faster than  $e^{-.5\alpha\theta}$  where  $\alpha$  can be smaller than one.

Expanding  $w(\theta)$  in terms of Laguerre polynomials (see Chapter 10), yields:

$$w(\theta) = \sum_{n=0}^{\infty} a_n L_n(\theta) \quad (102)$$

Where  $L_n(\theta)$  is the nth Laguerre polynomial, and:

$$\int_0^{\infty} e^{-\theta} L_n(\theta) L_{n'}(\theta) d\theta = \begin{cases} 0 & \text{if } n \neq n' \\ (n!)^2 & \text{if } n' = n \end{cases} \quad (103)$$

Therefore, coefficients of the expansion can be calculated from:

$$a_n = \int_0^{\infty} e^{-\theta} \frac{L_n(\theta) w(\theta) d\theta}{(n!)^2} = \int_0^{\infty} \frac{L_n(\theta) f(\theta) d\theta}{(n!)^2} = \frac{L_n(M)}{(n!)^2} \quad (104)$$

Where  $L_n(M)$  has the same form as  $L_n(\theta)$  except  $\theta^i$  is replaced by  $M_i$  (the  $i$ th moment of  $f(\theta)$ ).

Again, knowing the moments of  $f(\theta)$ , one can conceptually determine the coefficients of expansion for Laguerre polynomials. However, the

Laguerre expansion approximating  $f(\theta)$  obtained from this method will not be useful because of the following two reasons:

- 1) The approximated function obtained in this manner may go negative due to the zeros of the Laguerre polynomials. This deficiency can, to some extent, be eliminated if one scales the original variable such that the scaled variable is less than the zeros of Laguerre polynomials.
- 2) Although the approximated function obtained in this manner satisfies the restrictions that are imposed by moments, it may not necessarily correspond to the maximum entropy distribution.

Therefore, both techniques described above are deficient in some aspects. The technique developed from this study takes advantage of both approaches, however. This technique is detailed below:

- 1) Expand the maximum entropy functional form of  $w(\theta)$  in terms of its Taylor expansion around zero. That is:

$$w(\theta) = e^{\theta} \cdot f(\theta) = e^{\lambda_0} \left[ 1 + \sum_{i=1}^k \lambda_i^* \cdot \theta^i \right] , \quad (105)$$

where

$$\lambda_i^* = \begin{cases} 1 + \lambda_1 & \text{for } i=1 \\ \lambda_i & \text{for } i \neq 1 \end{cases} . \quad (106)$$

- 2) Rewrite the Laguerre expansion of  $w(\theta)$  in terms of polynomials of  $\theta^i$ , that is

$$w(\theta) = \sum d_i \theta^i = \underline{d}^T \underline{\theta} , \quad (107)$$

where  $\underline{d}^T$  is the transpose of  $\underline{d}$  which is a column vector con-

sisting of elements  $d_i$ . The exact form of  $d_i$  and the vector  $\underline{d}$  is described in Chapter 10.

- 3) Finally, equating the coefficients of the Taylor expansion of the Maximum Entropy Distribution, given by Eq. 105, with that of the Laguerre expansion given by Eq. 106, would yield unique values for the Lagrangean multipliers,  $\alpha^i$ , with no need of a complex iterative process.

To test the method, several applications for different types of distributions were performed. The results were generally satisfactory. Some of these test applications are discussed here.

1. Application to Exponential Distribution:

For the exponential distribution of the form:

$$f(x) = a \cdot e^{-ax} \quad , \quad (108)$$

the various moments can be generated by:

$$M_k = \frac{(k!)}{(a^k)} \quad . \quad (109)$$

To assure the convergence in Laguerre expansion, the program always performs at a minimum change of variable from  $x$  to  $u$  defined by:

$$u = \frac{x}{M_1} \quad . \quad (110)$$

Therefore,

$$f(u) = e^{-u} \quad . \quad (111)$$

For the new variable  $u$ , now the transformed moments can be calculated by:

$$M_k^u = \frac{M_k^x}{(M_1^x)^k} \quad , \quad (112)$$

where  $M_k^u$  and  $M_k^x$ , are the  $k$ th moments with respect to  $u$  and  $x$ , respectively.

Under this transformation any exponential function can be transformed to:

$$f(u) = e^{-u} \quad . \quad (113)$$

Therefore, it is only needed to investigate the estimates of the approximate function compare to  $e^{-u}$ . This was performed for varying numbers of moments. The results indicate that the approximate function estimated by this method exactly coincides with the actual function regardless of the number of moments. The result of this application is shown in Figure 3. It is to noted that "NM" stands for the number of moments, and the case associated with "NM= $\infty$ " is the graph of the actual distribution function.

## 2. Application to beta distribution:

The general form of a beta distribution is given by Eq. 114.

$$f(x) = \frac{\Gamma(a+b+2)}{\Gamma(a+1)\Gamma(b+1)} \cdot x^a \cdot (1-x)^b \quad \text{for } a, b > 0 \quad , \quad (114)$$

where  $\Gamma$  stands for gamma function defined by:

$$\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt \quad \text{for } z \text{ real and positive} \quad (115)$$

$$(z-1)! \quad \text{for } z \text{ integer and positive}$$

The moments of beta distribution can be obtained from Eq. 116.

$$M_r = \frac{\Gamma(2+a+b) \cdot \Gamma(a+1+r)}{\Gamma(a+1) \cdot \Gamma(a+b+2+r)} \quad . \quad (116)$$

The approximate function for a beta distribution using the method for various numbers of moments were estimated. The results are, in gen-

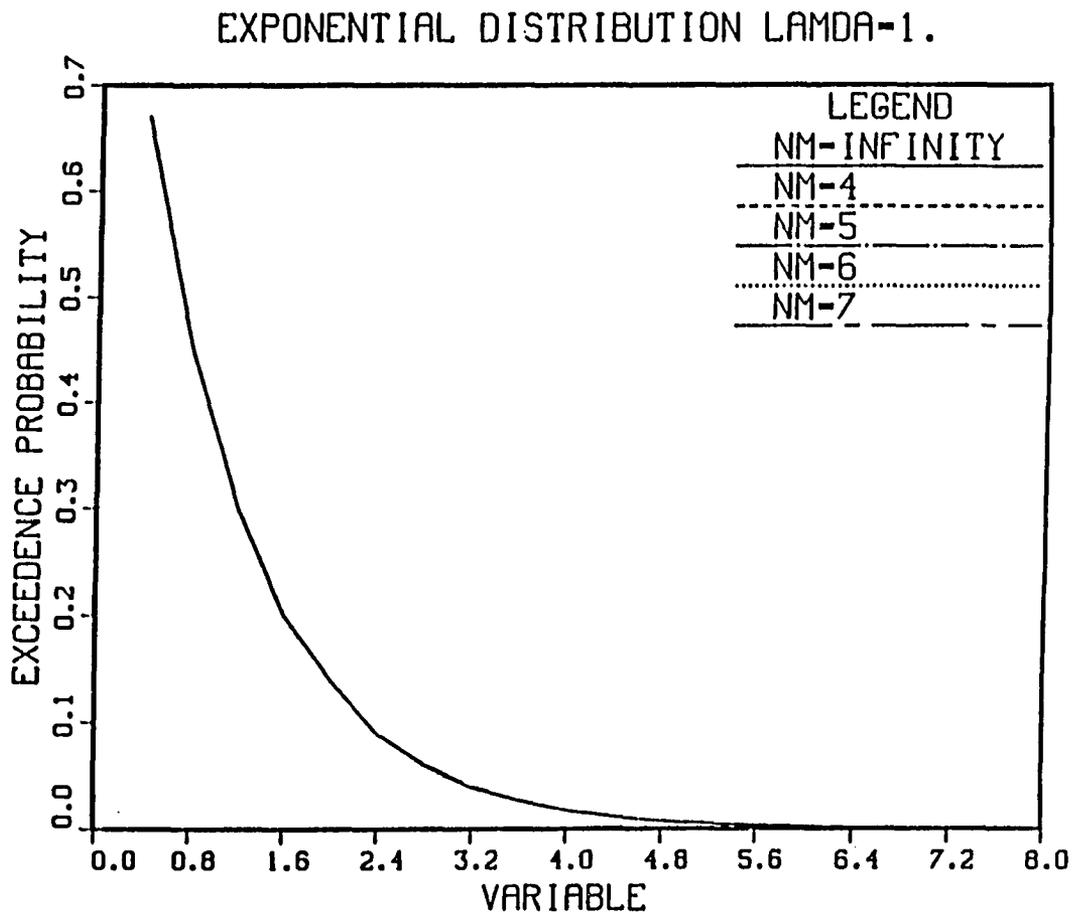


Figure 3. Maximum entropy approximation for exponential distribution

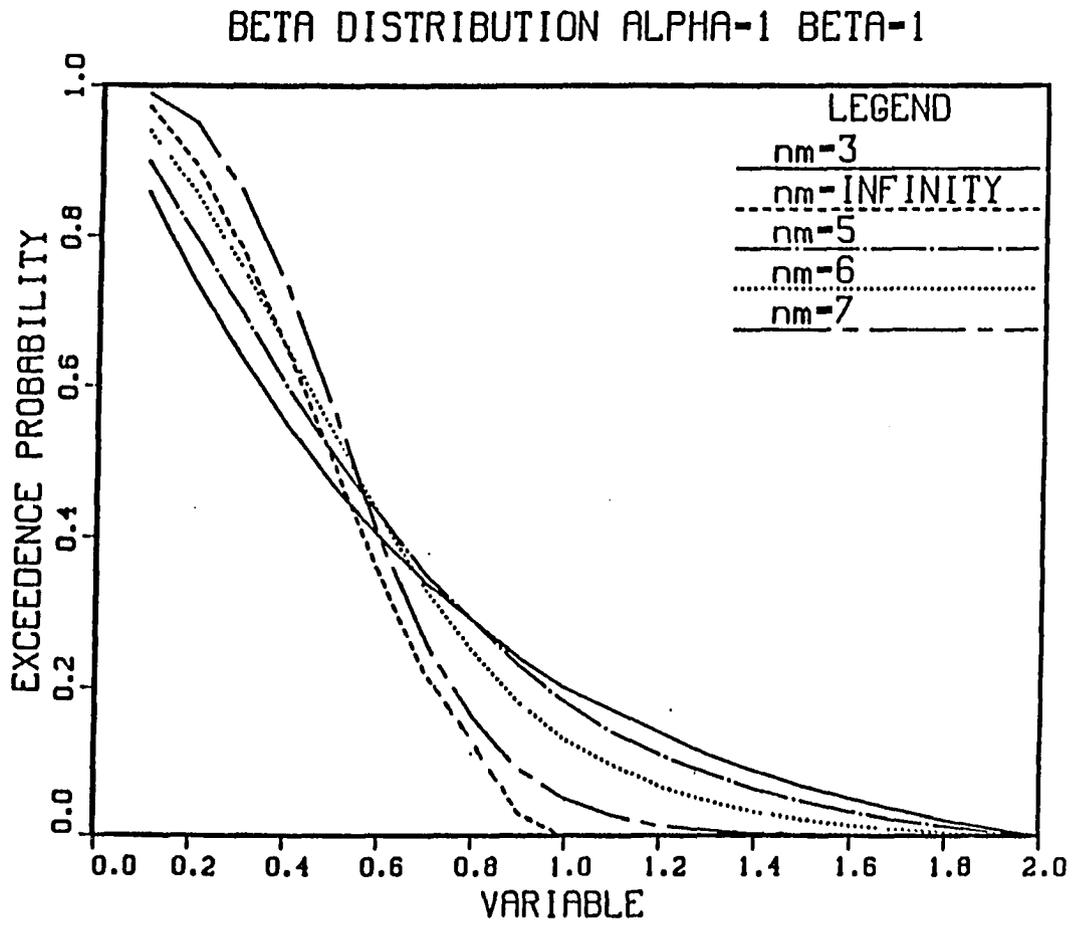


Figure 4. Maximum entropy approximation for beta distribution

eral, satisfactory when the number of moments is greater than five. Figure 4 depicts a typical result for the case when both a and b are equal to one. With seven moments, the accuracy of approximation in terms of deviation from the actual exceedance probability, is less than 3 percent.

### 3. Application to gamma distribution:

The general form of gamma distribution is given in Eq. 117.

$$f(x) = \frac{1}{b^a} \cdot \Gamma(a) \cdot x^{a-1} \cdot \frac{e^{-x}}{b} \quad (a > 0, b > 0) \quad (117)$$

The moments of a gamma distribution can be calculated from Eq. 118 as:

$$M_r = \begin{matrix} a \cdot b \\ a \cdot (a+1) \cdot (a+2) \cdot \dots \cdot (a+r-1) \cdot b^r \end{matrix} \quad \begin{matrix} r=1 \\ r \geq 2 \end{matrix} \quad (118)$$

The approximate function for a gamma distribution using method for various numbers of moments was estimated. In general, the results are satisfactory. Figure 5 depicts a typical result for the case when a and b were equal to two and one, respectively. The agreement between the approximate function and the actual one appears to be excellent.

It is important to note that in this case the Laguerre expansion of the transformed variable became negative for large values of x (due to the zeros of Laguerre polynomial as was discussed earlier). This, in turn, caused a poor estimation of the Lagrangean multipliers of the maximum entropy distribution. This problem was resolved simply by transforming the original variable "x" to a variable u defined by:

$$u = \frac{x}{k \cdot M_1} \quad , \quad (119)$$

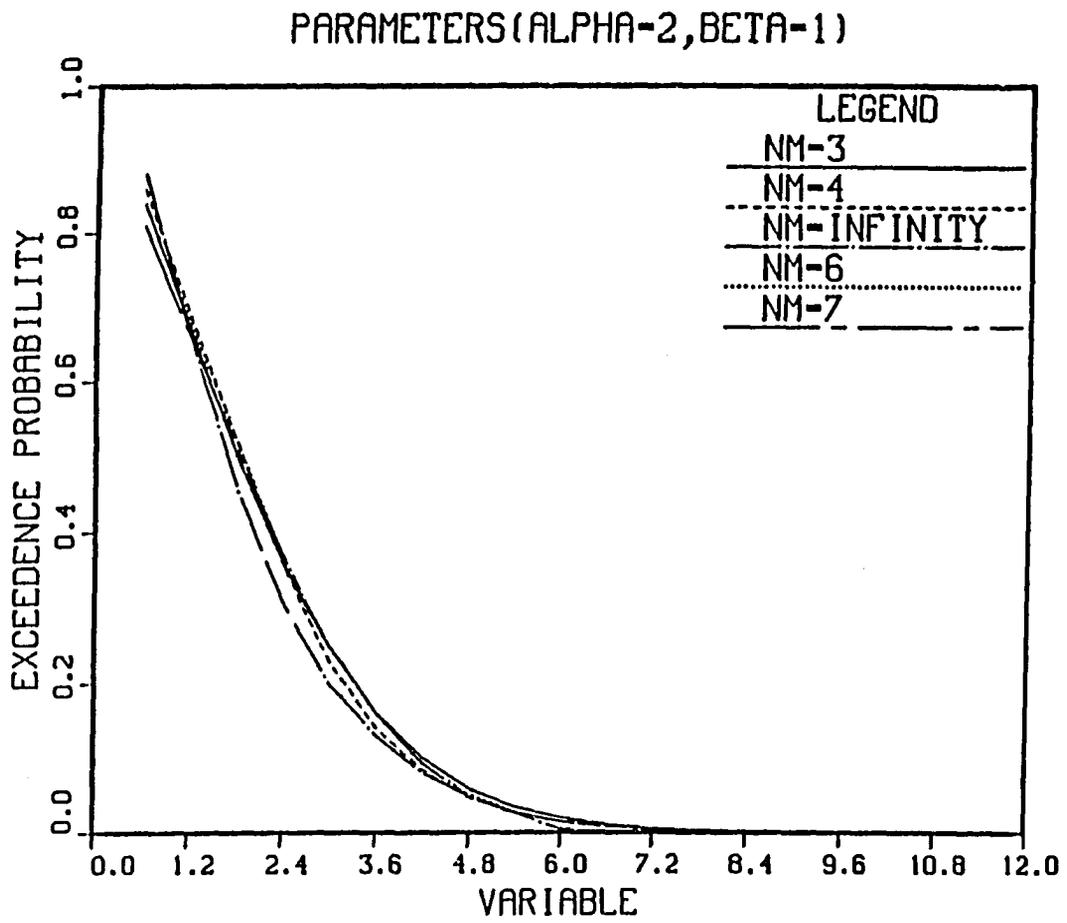


Figure 5. Maximum entropy approximation for gamma distribution

where  $k$  is automatically adjusted by the program such that to assure the Laguerre expansion of the distribution remains positive for all practical ranges of interest (up to 99th percentile of the estimated distribution).

#### 4.3. General Solutions When Several Moments Exist

In this case both uptime and downtime distributions are of general form and the analysts have only partial information regarding the underlying distributions. Typically, the available information is in the form of the knowledge about the first several moments of each distribution. These moments could have been generated based on the empirical distribution generated from the past observations.

Under this condition, the problem that an analyst is faced with is to determine the exceedance probability of cumulative down time using an alternating renewal process (as previously described by Eq. 86) when the only available information is the value for the first  $K$ -moments of downtime distribution ( $G$  or  $g$ ) and  $L$ -moments of uptime distribution ( $F$  or  $f$ ). Knowing the first  $K$  moments for any distribution  $f$ , one can calculate the first  $K$  moments of the  $n$ th convolution of  $f$  (usually designated by  $f_n$ ) using the following recursive equation:

$$M_j^{(n)} = \sum_{i=1}^j \binom{i-1}{j-1} M_{j-i+1} \cdot M_{i-1}^{(n)} \quad , \quad (120)$$

with

$$M_1^{(n)} = n \cdot M_1 \quad ,$$

where  $M_i^{(n)}$  is the  $i$ th moment of the  $n$ th convoluted function.

Knowing the moments of the  $n$ th convolution, one can use the technique described in the previous section to obtain the maximum entropy functional forms of  $F_n(t-d)$  and  $G_n(d)$ , therefore evaluating the cumulative downtime exceedance probability using Eq. 86.

This solution technique is computerized in the ACOT computer program developed for this dissertation. The solutions from this technique are compared to the actual solutions when the uptime distribution is exponential and the downtime distribution is beta in one case and exponential in the other case. The results are discussed in Chapter 5. Generally, the solutions obtained from this technique are in good agreement with the actual solutions as well as the approximate solutions that one can obtain from a compound Poisson process. In almost all cases, the solutions obtained using this technique bound the solutions from both the exact ones as well as the approximate ones obtained from the compound Poisson process. Bounding is defined here in terms of larger uncertainty or larger percentile for the tails of the distribution.

#### 4.4 General Solutions Under no Restriction

In this case, both the uptime and the downtime distributions are of general forms with no restriction attached. The analyst has either the functional form or the discrete values for the empirical distribution. The solution technique used for obtaining the cumulative downtime distribution in a general manner is through Monte Carlo simulation. The program written for this purpose called "Simulation". For a single component the program generates a sample time for the first downtime when the component goes down and then it generates another sample for the

duration of the downtime. The program repeats the same process for the second downtime, third downtime, so forth until the observation time "t" is reached. This is considered as one iteration and one sample can be generated for the cumulative downtime duration. If the process is repeated many times then an empirical distribution for the cumulative downtime can be generated. This empirical distribution approaches the actual distribution when the number of samples or iterations approaches infinity. The runs made for this study usually contains about one thousand iterations to assure high accuracy.

The simulation program is written so as to differentiate between the downtimes of different nature, e.g., a component can go down either because of catastrophic failure or minor repairs due to degradation. When the component goes down because of catastrophic failure its downtime duration comes from a different distribution than when the component goes down for minor repair. In addition, the simulation program can evaluate the cumulative downtime distribution within a system composed of many components using the straightforward Monte Carlo sampling process.

It is to be stressed that, even though the Monte Carlo simulation is an effective tool when dealing with a small number of components with a limited number of occurrences, it is usually impractical when used for a large number of components with large occurrence frequencies (i.e., due to the large amount of calculations involved and heavy computer cost). In addition, Monte Carlo samplings are unmanageable when dealing with dependencies as described in Chapter 3 under a marked Markov proc-

ess. In these cases the other solution techniques are far superior to Monte Carlo sampling.

Systems with a large number of states and large transition rates usually are found in statistical mechanics and reactor physics. In these cases, Monte Carlo sampling has a limited application and more reliance is placed on the other techniques.

For the intermediate system size (hundreds of components and hundreds of occurrences) Monte Carlo Sampling can be made more efficient by use of the stratified sampling techniques. One such sampling technique known as the Latin hypercube sampling was used as part of this study and is discussed in Chapter 11.

## 5. THE CAPABILITIES OF THE COMPUTER PROGRAMS AND THE COMPARISON OF THE SOLUTION TECHNIQUES

Preliminary investigation in determining the properties of ACOT distributions and their sensitivities to input variables are discussed in this chapter. The comparison of cumulative downtime distributions generated by various solution techniques which were described in Chapter 4, are also investigated here.

Section 5.1 summarizes the main features of the two computer programs that have been developed for this study. Section 5.2 discusses the behavior of cumulative downtime distribution and identifies its main contributors. Sections 5.3, 5.4, and 5.5 discuss comparisons among the various solution techniques (see Chapter 4) with that of the alternating renewal process (i.e., the exact solution).

### 5.1 Main Features and the Capabilities for ACOT and Simulation Programs

The ACOT program is written in Fortran V on VAX/VMS for the evaluation of the cumulative downtime distribution for a single component. The program models both the alternating renewal process and the compound Poisson process. For the alternating renewal process described earlier by Equation 86, the program also evaluates the truncation error generated by performing the summation on a limited number of indices (the first twenty terms are usually calculated by the program as a default). The program has the ability to treat ten different options for either the uptime or the downtime distribution. These options are normal, exponential, gamma, Cauchy, one-sided stable, mixture of two exponentials,

mixture of two normals, mixture of two gammas, generic distributions for compound Poisson process, and finally the specified number of moments for each distribution. For the final two options, the program uses the maximum entropy routine to determine a proper distribution based on moments as described earlier in Section 4.3.

The simulation program is also written in Fortran V on VAX/VMS and evaluates the cumulative downtime distribution through Monte Carlo simulations for single as well as multiple components. The program is written in modular format so that any distribution can be modeled by simply changing the associated module formula. The built in options of the program for the types of distributions are exponential, uniform, or tabular empirical. The program requires as input the uptime distribution for catastrophic failures, the uptime distribution for minor repairs (degraded failures), the distribution for undiscovered downtime when the component is in standby (a uniform distribution usually is used for this item), and finally repair duration or downtime distribution.

The flow chart as well as the listings of these programs with required inputs are described in Chapter 12.

## 5.2 Renewal Process-Single Component, Exact Solutions

Preliminary runs of the ACOT program were performed using the option of the alternating renewal process and those distribution for which compact convolution forms exists. The purpose of this preliminary investigation is to assess the effect of downtime frequency, as well as repair distribution (restoration duration) on the behavior of cumulative downtime distribution. Cumulative down time criteria usually are set at

high percentiles of the cumulative downtime distribution. Therefore, the behavior of the distribution tail is of main concern.

The main questions of interest in this preliminary investigation were:

1. What is the main contributor to the tail of cumulative downtime distribution, namely is it the large number of repairs or large restoration periods?
2. Given various shapes of repair distributions, such that mean and variance are conserved, which distribution would provide bounding results in the tail section of the cumulative downtime distribution?
3. What are the effects of the variance of repair distribution on the tail section of cumulative downtime distribution?

Figure 6 presents a typical graph of the sensitivity analyses that were performed to search for an answer to Question 1, above.

In these sensitivity studies the mean frequency of downtime occurrences (repair frequency) was varied and the repair distribution was kept fixed. The tails of the distributions were strongly affected by the frequency of the repairs as it is seen in Figure 6. On the contrary, when the distribution of downtime occurrences was kept constant and the mean of the repair distribution was changed, the effect on the tail of the cumulative downtime distribution was not significant, especially when the expected repair frequency within the time period was greater than one. In summary it appears that the tail of cumulative

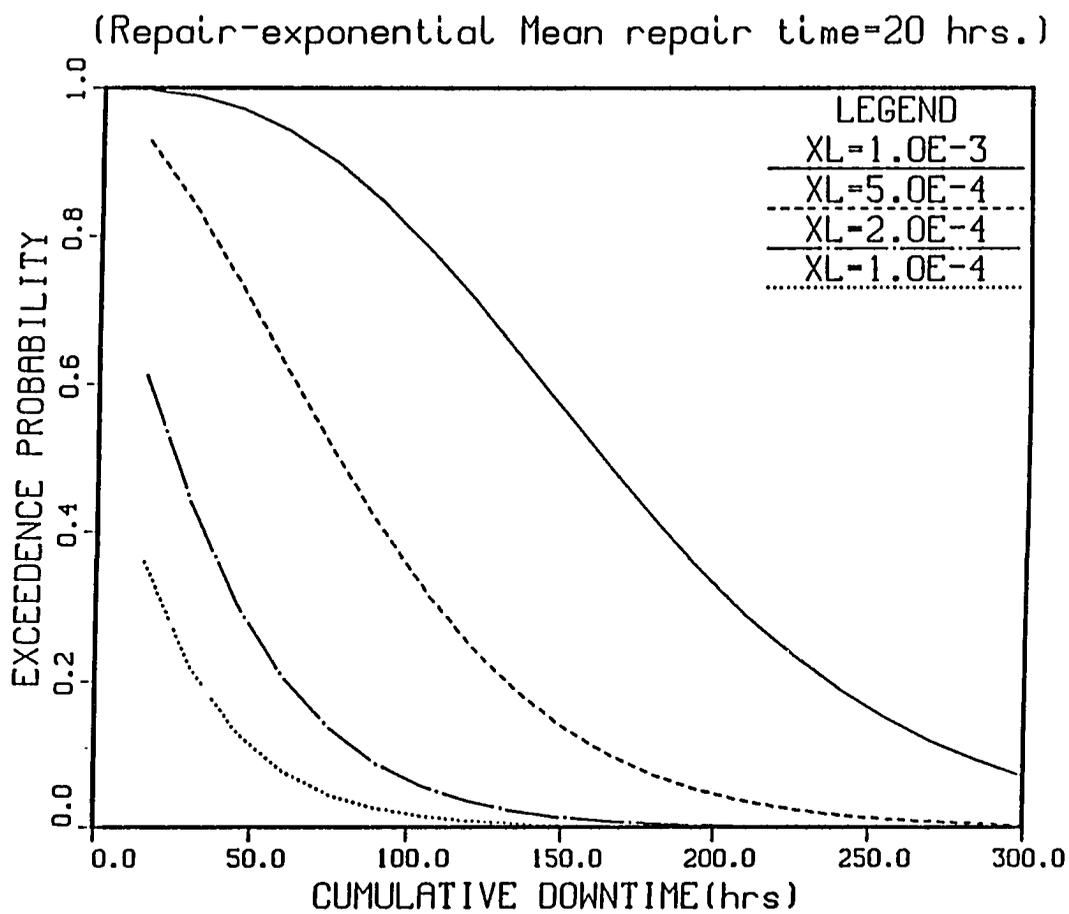


Figure 6. Effect of expected downtime frequency

downtime distribution is generated by the possibility of a large number of failures rather than large duration of few repairs. This conclusion was further substantiated by actually printing out the contributions of the first twenty terms of the summation in the alternating renewal equation for various cumulative downtimes and verifying that the tail values are mainly generated by the higher order terms.

Figures 7 and 8 present typical graphs of the sensitivity analyses that were performed to search for an answer to Question 2. In these sensitivity analyses, the distributions of the downtime occurrences as well as the mean and variance of the repair distribution were kept fixed. The shape of the repair distribution then was varied to cover all the possible situations (including bimodal distributions; mixture of normals, gammas, etc.). The results, shown in Figures 7 and 8, indicate that the selection of Exponential distribution for repair duration usually results in conservative bounding values for the tail of cumulative downtime distribution (response to Question 2).

Finally, Figure 9 presents a typical graph of the sensitivity analyses that were performed to search for a response to Question 3, namely the effect of the variance of repair distribution on the tail of the cumulative downtime distribution. No strong trend between variance and the behavior of the tail could have been identified. The main conclusion is that the effect of the repair distribution variance on the tail behavior of the cumulative downtime distribution is small and actually insignificant as long as the variance is varied within a factor of two.

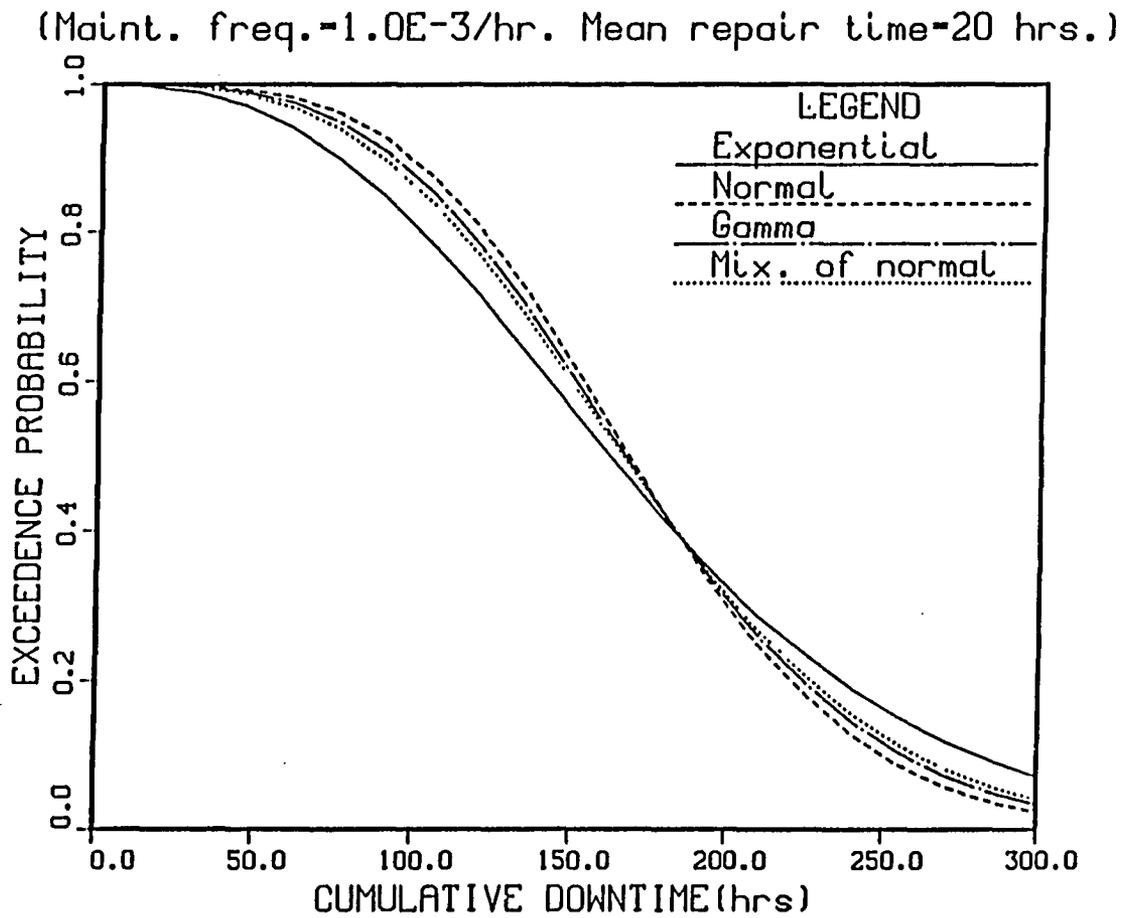


Figure 7. Effect of repair time distribution

(Maint. freq. =  $2.0E-4$ /hr. Mean repair time = 20 hrs.)

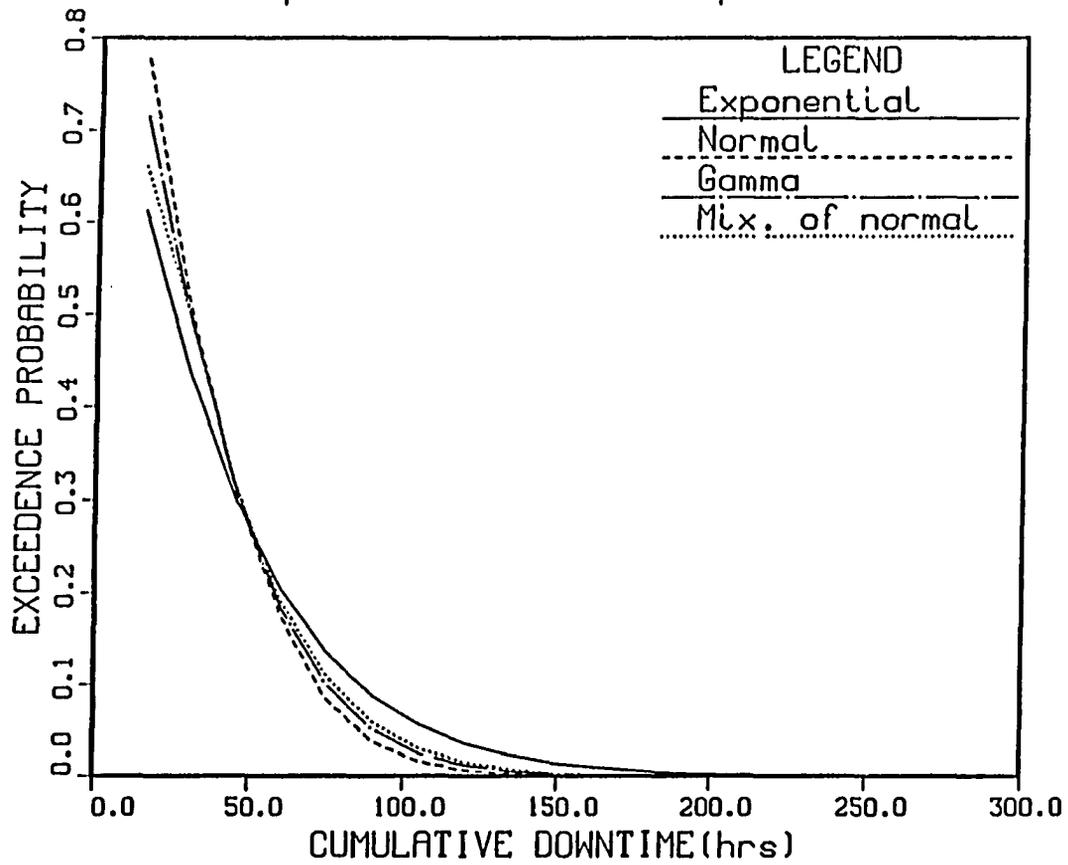


Figure 8. Effect of repair time distribution

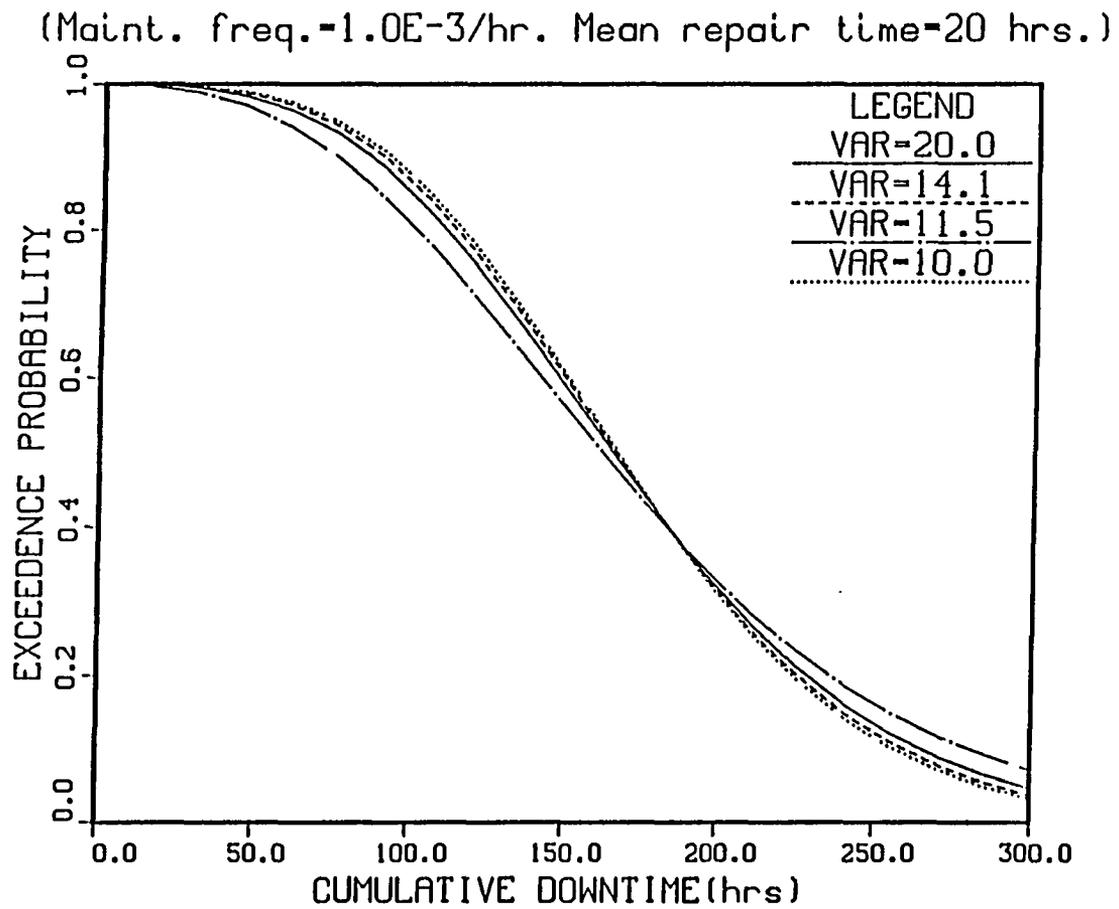


Figure 9. Effect of variance on repair time distribution

### 5.3 Compound Poisson Process—Single Component, Comparison with the Exact Solutions

Preliminary runs of the ACOT program were performed using the option of compound Poisson process. The purpose of this preliminary investigation was to assess: i) the conservatism in the compound Poisson process by comparing the estimated moments from this process with those obtained from the exact solution using the alternating renewal process, and ii) the conservatism which is introduced by transforming the moments into a distribution based on the maximum entropy principle using the techniques discussed in Section 4.2, and comparing this distribution with that obtained from the exact solution from the alternating renewal process. It is important to note that the conservatism here is considered only for the upper tail of the distribution where the interest of this study lies.

These assessments were performed through sensitivity studies. For the assessment of the conservatism in the compound Poisson process (the first of the two issues), both downtime and uptime distribution were considered to be exponential. The mean transition time for the uptime distribution was chosen to be one thousand hours (i.e., this translates to an expected 8.76 downtimes per year). The mean transition time for the downtime distribution (mean repair time) was varied from one hour to one hundred hours. The first two moments and the variance of the actual solutions with those estimated from the compound Poisson process for the cumulative downtime distribution in an overall period of one year are tabulated and provided in Table 6. As can be seen the error generated

Table 6. Comparison of the Moments Calculated by Compound Poisson Process vs Alternating Renewal Process

Mean Repair Time	Compound Poisson Process			Alternating Renewal Process		
	First Moment	Second Moment	Variance	First Moment	Second Moment	Variance
1.00E+00	8.76E+00	9.43E+01	4.18E+00	8.75E+00	9.40E+01	4.15E+00
5.00E+00	4.38E+01	2.36E+03	2.09E+01	4.37E+01	2.30E+03	1.98E+01
1.00E+01	8.76E+01	9.43E+03	4.19E+01	8.70E+01	9.20E+03	4.04E+01
2.00E+01	1.75E+02	3.77E+04	8.37E+01	1.70E+02	3.56E+04	8.18E+01
5.00E+01	4.38E+02	2.36E+05	2.09E+02	4.10E+02	2.10E+05	1.95E+02
1.00E+02	8.76E+02	9.43E+05	4.19E+02	7.90E+02	7.50E+05	3.55E+02

Note: Mean uptime is 1000 hours and total period is one year.

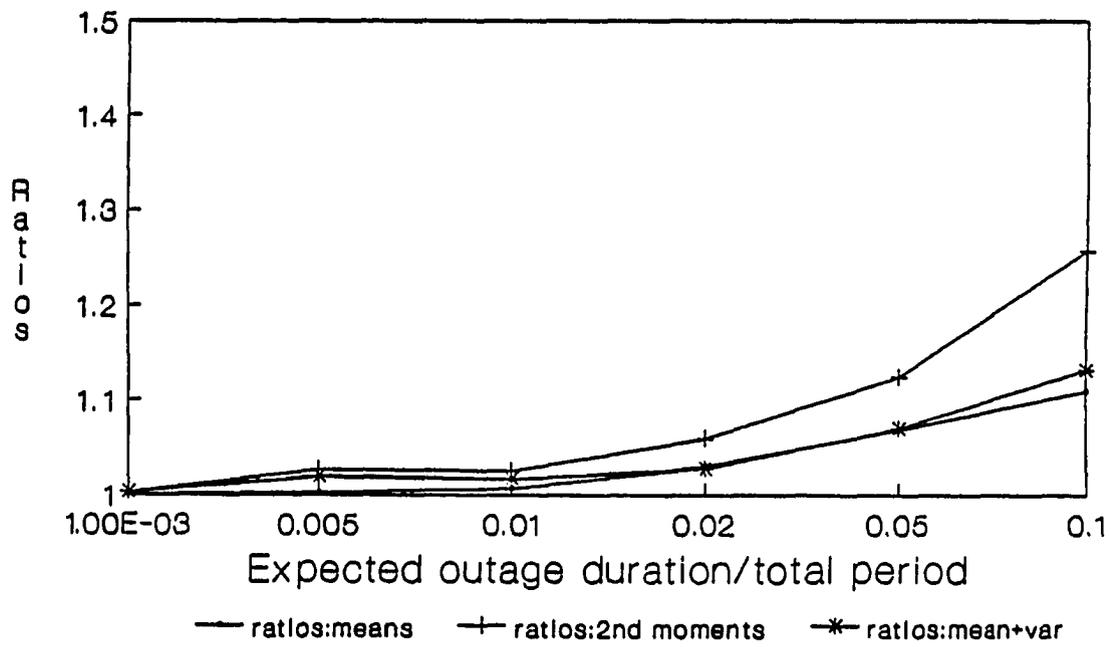


Figure 10. Comparison of compound vs alternating renewal processes; ratios of mean, 2nd moment and mean+var

by using a compound Poisson process is less than five percent for the cases where the ratio of the expected cumulative downtime over the total period is less than 0.02 (i.e., when the expected cumulative downtime is less than 180 hrs in one year). These results are graphically shown in Figure 10. In this figure, the ratio of the actual over the estimated values for the first moment, the second moment and the mean plus variance are graphed against the ratio of the expected downtime over the total period.

To assess the conservatism generated by the distribution fitting procedure described in Section 4.2, two sensitivity runs were performed. In both of these runs, the uptime distribution was taken as exponential with a mean transition time of one thousand hours. The downtime distribution was also considered to be exponential. Two sensitivity cases, corresponding to mean repair time of one hour and ten hours, were performed. These values are chosen such that to ensure that the conservatism generated by the compound Poisson process itself is minimal. The distribution fitting was performed based on two, four, and six moments for each of the above two cases. The results are graphically presented in Figures 11 and 12. The conservatisms that are illustrated by these figures through comparing the actual vs the approximate results mainly stems from two sources, i) that portion of the conservatism that is inherent in a maximum entropy approach for distribution fitting, and ii) that portion of the conservatism that is generated from the specific procedure used to obtain the maximum entropy fit (this technique or fitting procedure was discussed in Section 4.2). A detailed discussion of

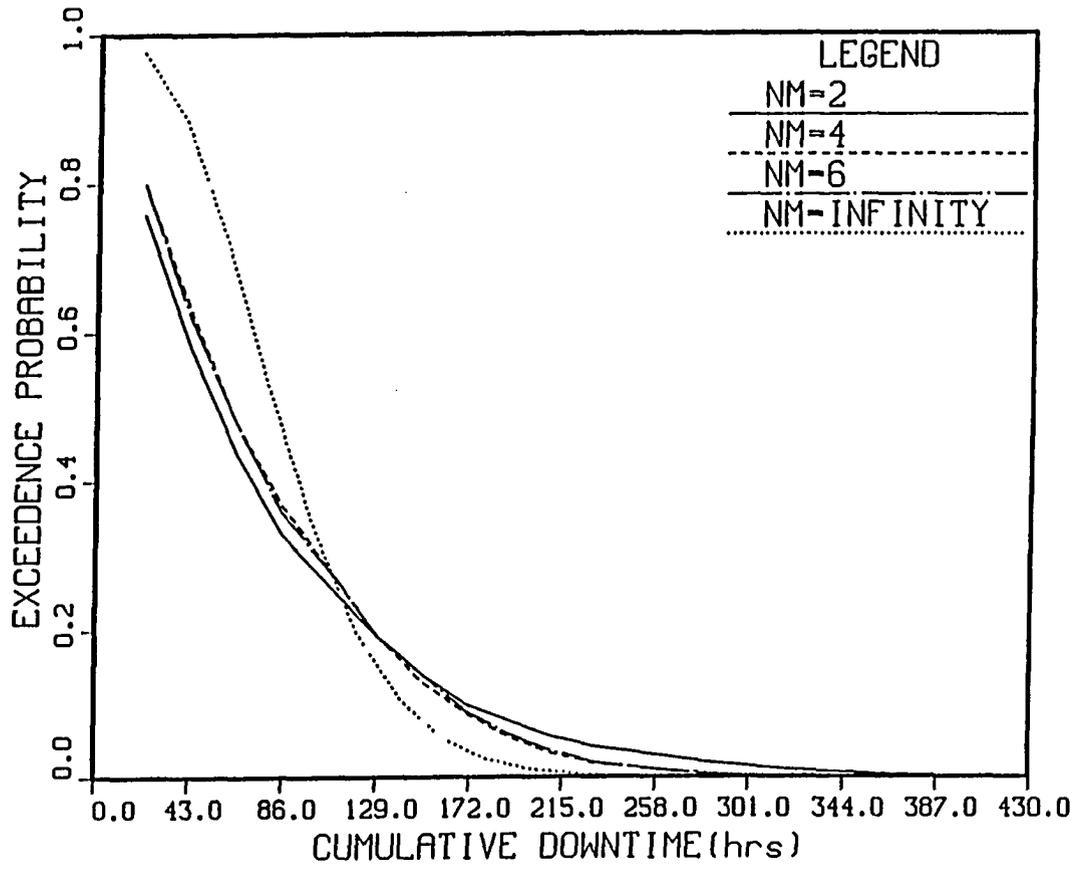


Figure 11. Comparison of compound Poisson process with alternating renewal process ( $MUT=1000$  hrs,  $MRT=10$  hrs)

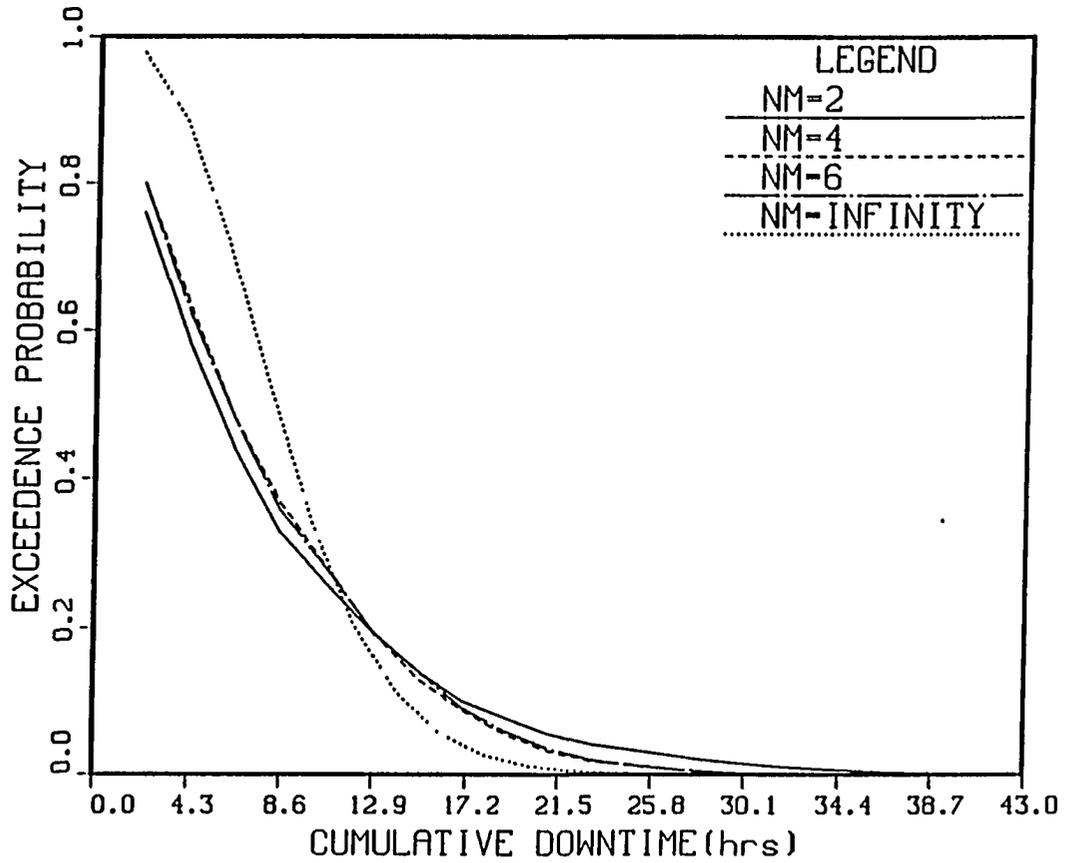


Figure 12. Comparison of compound Poisson process with alternating renewal process (MDT=1000 hrs, MRT=1 hr)

the conservatism that can be generated by the distribution fitting technique and some remedies to minimize this conservatism is discussed in Chapter 10.

Generally, the following conclusions can be made regarding the overall conservatisms in the estimated tail of the distribution of cumulative downtime:

- i) There is not much incentive to specify more than four moments for the downtime distribution when using the compound Poisson process, and
- ii) The overall error of conservatism using the compound Poisson process with the maximum entropy fit for the cumulative downtime distribution at ninety-five percentile is about twenty-seven percent for six moments, twenty-nine percent for four moments, and sixty-four percent for two moments.

#### 5.4 Moment Method-Single Component, Comparison With The Exact Solutions

The moment method was described earlier in Section 4.3. In general, the moment method is applicable when the information available is knowledge about the first several moments of uptime and downtime distributions with no knowledge about the distribution itself. The ACOT program then creates distributions based on moments for all the multiple convolutions of uptime and downtime distributions using the maximum entropy principle. The program then uses the alternating renewal process to generate the cumulative downtime distribution.

To assess the adequacy of this method, three sensitivity runs were performed. In all of these sensitivity runs, the uptime and the down-

time distributions were taken to be exponentials with mean transition times of one thousand hours and one hour respectively. The three sensitivity runs that were performed are describe below:

Sensitivity Run 1: The exact form of uptime distribution was specified but the downtime distribution was specified by its first two moments.

Sensitivity Run 2: The first two moments were specified for both uptime and downtime distributions.

Sensitivity Run 3: The first three moments were specified for both uptime and downtime distributions.

The results of these three sensitivity runs along with the exact solution obtained from specifying both uptime and downtime distributions are depicted in Figure 13. The following conclusions can be obtained from these graphical results:

- 1) If the exact distribution of uptime is known, the method would provide a good approximation of cumulative downtime distribution as long as a minimum of two moments for downtime distribution is known. Therefore this method is superior to the compound Poisson process described earlier. A simple comparison of the errors at a ninety-five percentile of cumulative downtime distribution indicates that this method yields an error of 25.5 percent compare to 64 percent from the compound Poisson process (note that these errors are associated with the case when only two moments of downtime distribution are known). In

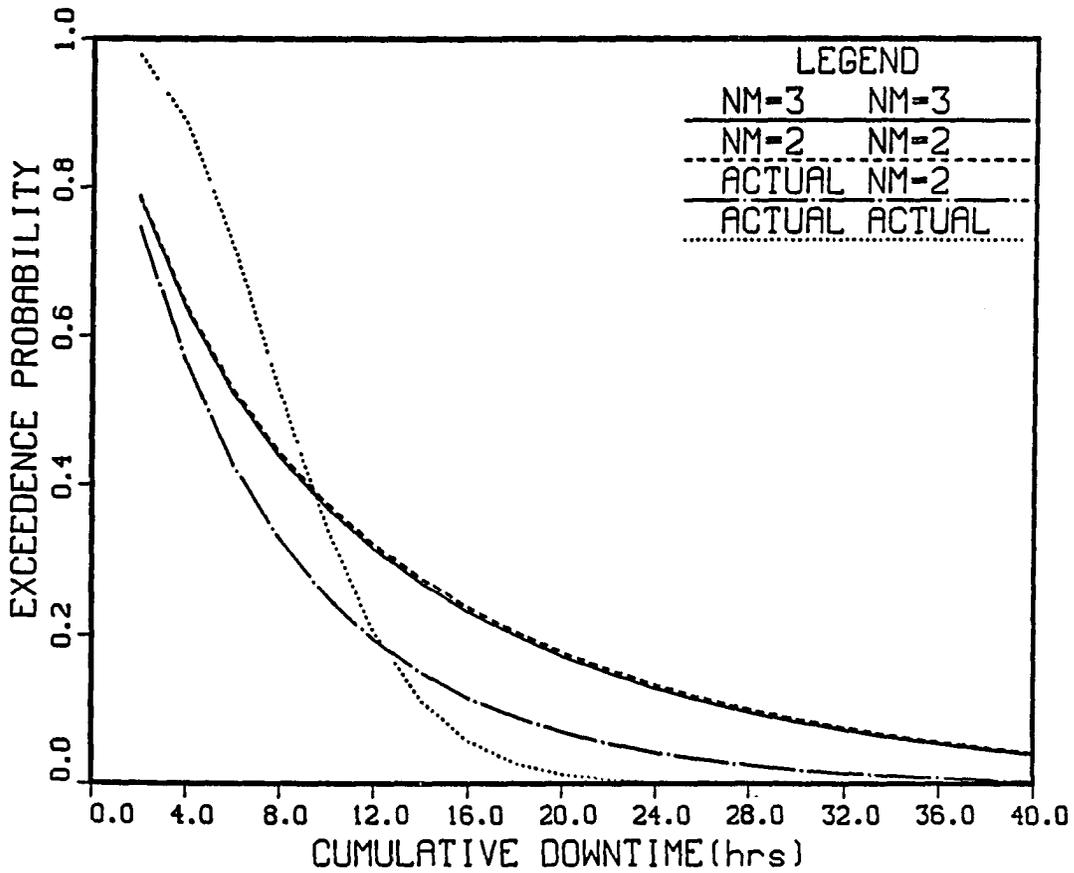


Figure 13. Comparison of moment method vs alternating renewal method (exponential, MDT=1000 hrs, MRT=1 hr)

addition, this method, unlike the compound Poisson process, does not impose any restriction on the uptime distribution.

- ii) The method provides a poor approximation to the tail of the cumulative downtime distribution when the exact form of uptime distribution is not known but the first few moments are specified. The estimated ninety-five percentile of the cumulative downtime distribution is higher than its actual value by a factor of two for three moments and a factor of 2.25 for two moments.

According to the above discussion, to accurately estimate the cumulative downtime distribution every attempt is to be made to determine the exact form of the uptime distribution. The form of the downtime distribution is of lesser importance and usually the determination of the first two or three moments would be sufficient for an accurate estimate.

#### 5.5 Simulation—Single Component, Comparison With The Exact Solution

The simulation technique for estimating the cumulative downtime distribution was briefly discussed in Section 4.4. The simulation program written for this purpose is also described in detail as a part of Chapter 12. The main concept in the simulation program is to generate waiting times for downtime occurrences from the uptime distribution, and repair times (downtimes) from the downtime distribution using a simple (non-stratified) Monte Carlo sampling. Each iteration stops when the summation of waiting times and downtimes equals or exceeds the total time period. At each complete iteration, a sample is properly generated

for the cumulative downtime distribution. The program, as default, generates one thousand sample points. The program can also generate simultaneous sampling of several components (present limit is 10) and estimate the cumulative downtime distribution of each or all of these components.

An expanded version of this program (not yet published) was also developed by this author as a part of an ongoing project at Brookhaven National Laboratory sponsored by Nuclear Regulatory Commission (entitled Risk-Based Performance Indicators). This expanded program can simulate several uptime and downtime distributions for a single component (due to various types of failures such as catastrophic, degraded, etc.). In addition to cumulative downtime distributions this program can also estimate the distribution of the unavailabilities of components, systems, and functions.

An illustrative simulation result is shown in Figure 14. In this simulation both the uptime and the downtime distribution are exponentials with mean transition times of one thousand hours and one hour respectively. The exact solution from the alternating renewal process is graphed along with four Monte Carlo simulations, each with one thousand trials, to show the expected variations of the Monte Carlo simulations. The error estimated at the ninety-five percentile of the distribution, measured by estimated variance from twenty runs each with one thousand trials, is less than two percent. This indicates that Monte Carlo simulation is a viable method of estimating the cumulative downtime distribution when all the needed distribution are well defined in terms of their actual distribution functions.

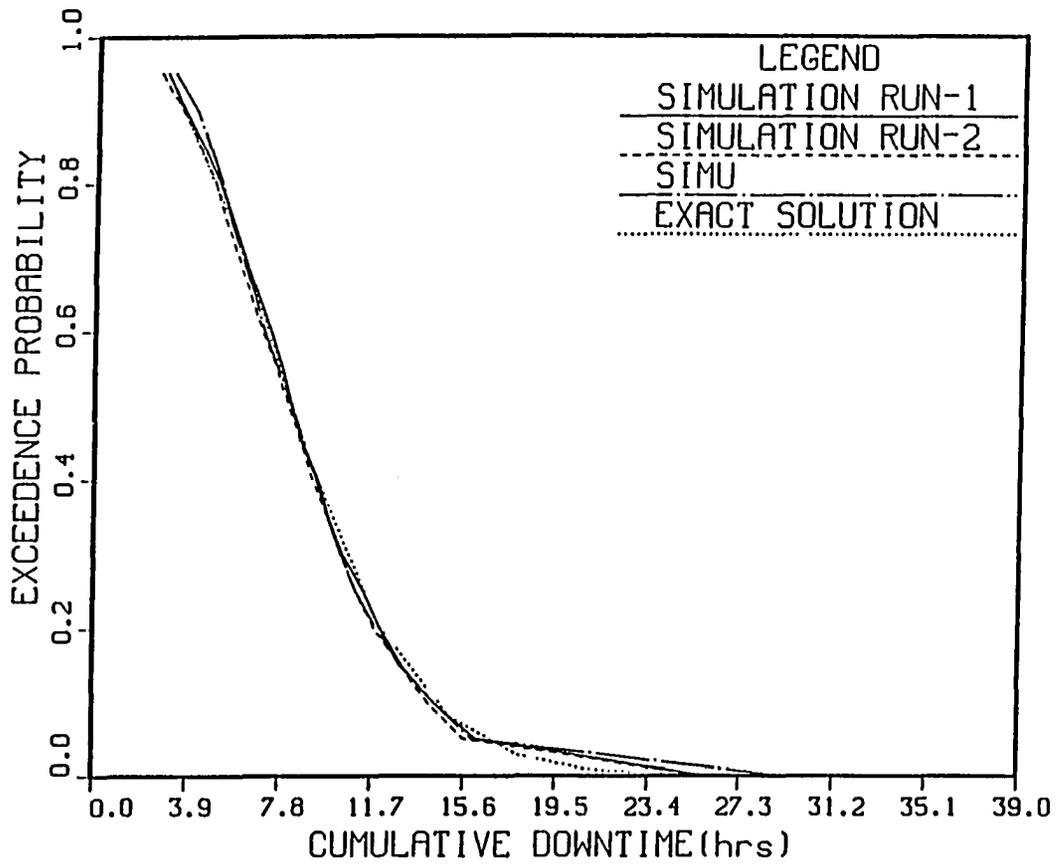


Figure 14. Variations in Monte Carlo simulations uptime and downtime both exponentials (MDT=1000 hrs, MRT=1 hr)

## 6. CUMULATIVE DOWNTIME DISTRIBUTION FOR A MULTI-COMPONENT SYSTEM

The cumulative downtime distribution for a single component was described in previous chapters. In this chapter, the concept and the methods for evaluations will be extended to address a multi-component system.

The system is defined here in a very general manner. The system can be of any logic structure and may contain any number of components. In general, the components may be different from one another. The cumulative downtime for all the system components in some time period is simply the sum of the cumulative downtimes of the individual components in that period. That is;

$$d_s = \sum_{i=1}^n d_i \quad , \quad (121)$$

where  $d_s$  is the system cumulative downtime,  $d_i$  is the cumulative downtime of component "i", and  $n$  is the total numbers of the components in the system.

The distribution of the cumulative downtime for the system ( $d_s$ ) can be written as the convolution of the cumulative downtime distributions of the individual components. There are generally four different methods for estimating the distribution of  $d_s$ . These are:

- i) Numerical Convolution: Standard numerical techniques for convolutions can be used to obtain the cumulative downtime distribution of all the system components. This is particularly efficient if there are relatively few components and the distri-

butions are discretized with relatively few points. It is, however, understood that this method not only introduces errors associated with the numerical integration routines but also it suffers from the additional errors generated by discretizing the distribution into few points. Controlling the effect of the aggregate errors on the tail of the cumulative downtime distribution is by itself a challenging task.

Another approach discussed by Keilson and Nunn<sup>12</sup> deals with Laguerre transformation as a tool for the numerical solution of the convolution integrals. This approach was not implemented in this dissertation, therefore its applicability can not be determined. The main concern in using the Laguerre transformation when one deals with the probability distribution functions is to assure that the function generated by the Laguerre expansion stays non-negative. It is not known how many moments are to be specified for a given function to assure the Laguerre expansion stays positive at least for the ranges of interest.

ii) Monte Carlo Simulation: Standard techniques of Monte Carlo simulations can be used to obtain the cumulative downtime distribution of a system composed of several components. A simple Monte Carlo technique is efficient and fast running in terms of computer execution time for relatively few components. The accuracy or the variance of the estimator can be controlled with the number of trials as discussed previously. For the large number of components, various variance reduction techniques and/or stratified samplings are available to assure accurate

results with relatively moderate amount of computer time. A brief discussion of these techniques is provided in Chapter 11.

iii) Moment Propagation: The various moments of the system cumulative downtime distribution can be generated from the moments of the individual component cumulative downtime distributions. The recursive equation for this purpose was developed and discussed in Chapter 4 which facilitates the use of the computer for generating these moments. These moments then can be fed to a maximum entropy routine (either an iterative routine or a routine similar to what was discussed earlier in Chapter 4) to obtain the distribution itself.

iv) Approximate Technique: The cumulative downtime distribution of a system as described earlier by Equation 121 is in the form of summation of  $n$  random variables. This distribution approaches a normal distribution according to the central limit theorem for large values of  $n$ . This gives the temptation of using the various approximations associated with the central limit theorem, known as the local limit theorem. One approach to the problem of near normality is to make a small correction to the normal distribution approximation by using the asymptotic expansions (known as Edgeworth or Gram Charlier) based on the central limit theorem. Due to the problems associated with the Edgeworth approximation (see Chapter 2 on the literature review) and also due to our interest in accurately estimating the tail of the distribution, the use of Escher's large deviation

technique (the various techniques of near normality are discussed in Chapter 11) appears to be promising.

In addition to the above techniques for estimating the cumulative downtime distribution of a system, there is another technique which only applies to the compound Poisson process. This technique was described earlier in Section 3.1 by Equations 14 and 15.

The remainder of this chapter compares the results of various techniques discussed here on a trial basis, excluding those techniques discussed in i) and iv) above. The numerical convolution techniques (item i above) are excluded since no additional insights will be gained from their examinations. The approximate solutions (item iv above), even though they are very interesting technically, are excluded from this trial application mainly because these techniques have been previously studied by several others. These techniques are discussed in detail in Chapter 11; however their applications to the determination of cumulative downtime distribution of multi-component system are left for future work.

#### 6.1 Distribution of Cumulative Downtime Risk For a Multi-Component System

Cumulative downtime risk for a multi-component system is defined by:

$$r_s = \sum r_i \cdot d_i \quad , \quad (122)$$

where  $r_s$  is the cumulative downtime risk,  $r_i$  is the risk incurred by the plant when the component "i" is down for a unit of time, and  $d_i$  is the cumulative downtime for component "i".

The quantity  $r_i$  in Probabilistic Risk Analyses (PRA) usually is called the risk impact measure or the importance measure of component "i". This quantity is simply calculated in terms of conditional probability of core damage or the conditional expectation of public risk (man rem exposure) when component "i" is not available. The various types of importance measures and their applications are discussed in Ref. 24.

There are similar problems in other areas of engineering. Generally, in any problem where a cumulative quantity stems from several sources with different magnitudes (or impacts), Equation 122 would be applicable.

The solution techniques for estimating the distribution of cumulative downtime risk are similar to those discussed in the previous section. Note that Equation 122 is equivalent to Equation 121 under the following transformation;

$$U_i = r_i \cdot d_i \quad \text{for } i=1, \dots, n \quad . \quad (123)$$

For a compound Poisson process the equations described in Section 3.1 (Equations 14 and 15) would be applicable if the repair distribution (actually the cumulative distribution of a single repair) of component "i" is scaled by the importance measure of component "i", namely  $r_i$ .

## 6.2 Trial Application/System Description

The system selected for this trial application is the Auxiliary Feed Water System (AFWS) at the Surry nuclear power plant. The function of the AFWS is to provide feed water to the secondary side of the steam generators upon loss of main feedwater. AFWS is composed of three trains, two of which are driven by electric pumps with a capacity of 350

gpm (gallons per minute), and one train is turbine driven with a capacity of 700 gpm. The decay heat can be removed from the primary system by providing at least 350 gpm of cooling flow through the secondary side of one or more operating steam generators (there are three steam generators in this plant). Therefore, the successful operation of the AFWS requires the operability of at least one train.

The active components in each train of the AFWS consists of a pump and two discharge valves. It shall be noted that the passive components such as the check valves, manual valves, strainers, etc., were not considered for this pilot application because of their insignificant contributions to cumulative downtime distribution. We also do not consider the discharge valves because they are shared by all the trains, so that the downtime of one valve train would give an insignificant impact on the operational reliability of the system.

Two more aspects of the problem needs to be discussed before the actual modeling is presented. These are:

- a) Considerations for standby components: The AFWS is a standby system, that is it is not normally operating. The operability of such components are verified during periodic operational testing of each component. The time interval between these operational tests, known in plant technical specifications as Surveillance Test Interval (STI), is every 31 days. Therefore, if a component within this system is found failed during operational testing, that component might have been out for a maximum period of 31 days before it was detected to be inoperable.

The actual downtime of that component would be its repair time plus the period it was out prior to the repair (this period is known as the undetected downtime). It is easy to show that under the assumption of exponential distribution for uptime, the mean undetected downtime would be one half the test interval (for this case 15.5 days). The undetected downtime can be accounted for by shifting the repair distribution by one half the test interval.

It is intuitively easy to understand that for a standby component, if the undetected downtime is significantly larger than the mean repair time, then the contribution of repair time to the component cumulative downtime would be negligible. Even though this is a very disturbing notion, since the operational test intervals are comparatively large (about a month), one should remember that there are a variety of other means to verify the component operability in addition to periodic operational testing. These are:

- i) Actual demands on the components due to plant transients which require the components to operate,
- ii) Daily walk through inspections of the components in safety systems which enables the utility staff to detect many types of failures such as minor leaks, wrong position of the valves or breakers, etc.
- iii) Indicator lights for the power supplies of the active components such as motor operated valves, electric pumps, etc.,

which allow the detection of failures which have resulted in de-energization of the components.

- iv) Various indicators for pressure, temperature, inventories, etc., which are monitored regularly to assure the component's operability. As an example, a level indicator on the condensate storage tanks enables the operator to detect any minor leaks resulting in a level drop.

If one considers all of the above supplementary ways of detecting component failures, one can usually justify an estimate for undetected component downtime much less than one half the test interval. The estimate of undetected component downtime for this trial application was assumed to be three and half days instead of one half test interval (15.5 days), due to the above mentioned considerations.

In addition to repair of failures, the components can also be repaired for minor faults (degraded failures). For the minor fault (degraded failures) the component is assumed to be operable prior to repair. Therefore, the undetected downtime does not apply.

- b) Consideration for common cause failures: The two electric driven pumps of the AFWS can be rendered inoperable by a single cause. For example, a high humidity in the room (which might have been caused by a crack in the steam line connected to the turbine driven pump) can cause dependent failures of both pumps. This type of failure, where multiple components fail simultaneously, is known as a common-cause failure. The common-cause failure must be evaluated separately if one desires

to estimate the cumulative downtime risk, simply because of the risk importance measure of two trains being down at the same time is much greater than when one train is down. For this application, common-cause failures are treated like single component failures with a specific occurrence rate, and the undetected downtime is assumed to be one half the test interval. This is a valid assumption as long as the periodic tests are performed sequentially (for staggered testing one fourth of the test interval is more suitable).

Table 7 provides the input information needed for estimating the cumulative downtime distribution and the distribution of the cumulative downtime risk.

The results obtained for this trial application are discussed in the following sections.

### 6.3 Cumulative Downtime and Cumulative Downtime Risk for "AFWS"

The solution methods for estimating the cumulative downtime and the cumulative downtime risk distributions were discussed earlier in this Chapter. Out of the five different methods described earlier, three were actually implemented for this trial application. These are, Monte Carlo simulation, moment propagation, and compound Poisson process. Furthermore, the moment propagation method presented in the form of maximum entropy distribution would yield essentially the same results as the compound Poisson process. This is because of the moments generated from the two processes are approximately the same. Therefore, for the remainder of this chapter, the comparisons are provided for various dis-

Table 7. Information Needs for Evaluating Cumulative Downtime Distribution and the Distribution of Cumulative Downtime Risk for Surry AFWS

AFWS Component	Information on <sup>a</sup> Uptime Dist.	Information on <sup>a</sup> Downtime Dist.	Importance <sup>b</sup> Measure
Failure of motor driven pump	Exponential with occurrence rate=2.2E-5/hr	Exponential with mean repair time of 18 hrs	1.2E-7
Degraded failure of motor driven pump	Exponential with occurrence rate=8.8E-5/hr	Exponential with mean repair time of 6 hrs	1.2E-7
Failure of turbine driven pump	Exponential with occurrence rate=8.0E-05/hr	Double exponential with parameters P <sub>1</sub> =0.5 MRT <sub>1</sub> =10 hrs P <sub>2</sub> =0.5 MRT <sub>2</sub> =26 hrs <sup>c</sup>	1.4E-7
Degraded failure of turbine driven pump	Exponential with occurrence rate=3.2E-4	Exponential with mean repair time of 20 hrs	1.4E-7
Common cause of two motor driven pumps	Exponential with occurrence rate=2.2E-6	Exponential with mean repair time of 26 hrs	1.8E-6

<sup>a</sup>The information for uptime and downtime distribution are based on a crude inferences made on operational data provided in Oconee PRA; Reference 25.

<sup>b</sup>The importance measures are derived from a simplified generic PRA model of Surry plant as described in Reference 24. The risk incurred is in terms of core-melt frequency per hour.

<sup>c</sup>The second distribution is shifted in time by two hours.

tributions of the cumulative downtime and the cumulative downtime risk obtained from the processes: the compound Poisson process coupled with the maximum entropy routine, and the alternating renewal process coupled with Monte Carlo simulations.

These comparisons are made in four categories. These are:

- a) Comparison of the cumulative downtime distribution obtained from the two methods when the undetected times are not accounted for. This comparison is referred to as the Case 1 comparison and is discussed in Section 6.3.1.
- b) Comparison of the cumulative downtime risk distribution obtained from the two methods when the undetected downtimes are not accounted for. This comparison is referred to as the Case 2 comparison and is discussed in Section 6.3.2.
- c) Similar to case "a" presented above except the undetected downtimes are accounted for. This comparison is referred to as the Case 3 comparison and is discussed in Section 6.3.3.
- d) Similar to case "b" presented above except the undetected downtimes are accounted for. This comparison is referred to as the Case 4 comparison and is discussed in Section 6.3.4.

It shall be noted that the time period used for the above evaluations is three years for all the cases.

#### 6.3.1 Case 1 Comparison

In this comparison the cumulative downtime distribution was estimated from both the alternating renewal process coupled with the Monte Carlo simulation and the compound Poisson process coupled with the maxi-

mum entropy routine. The contributions of the undetected downtimes have not been considered for this case. Table 8 presents the input data needed for using the compound Poisson process for this trial application. These data are calculated directly from the input information discussed in Section 6.2 and presented in Table 7 using the process described earlier.

The graphical result for Case 1 comparison is provided in Figure 15. The cumulative downtime distribution calculated from the compound Poisson process uses only the first two moments for the repair distribution. As was demonstrated earlier in this study, if higher moments were specified for the repair distribution the agreement between the two curves would be improved. However, in almost all practical applications of these techniques, the knowledge about the repair distribution would be limited to the first two moments. It also should be noted that the main interest in these applications is to estimate the tail of the distribution (i.e., exceedance probabilities smaller than 0.05). This is the region where the two graphs are in good agreement. The fact that the compound Poisson process and the maximum entropy fitting technique always bound the actual distribution is also of great importance. In any regulatory application the decision is to be made in a conservative manner while at the same time take advantage of all existing information. This description is in agreement with the results obtained from the compound Poisson process.

According to Figure 15, the probability that the cumulative repair downtimes of all the components in AFWS exceeds 550 hrs is less than

Table 8. The Input Information Generated From Table 7 for the Application of Compound Poisson Process to AFWS

DEFINITION OF RUNS	UPTIME DISTRIBUTION	DOWNTIME OR DOWNTIME RISK DISTRIBUTION
cumulative downtimes excluding undetected downtimes	Exponential with $\Lambda = 6.222E-4$	General with two moments M1=15.92 M2=596.1
cumulative downtime risk excluding the undetected downtimes	Exponential with $\Lambda = 6.222E-4$	General with two moments M1=2.322E-6 M2=2.67E-11
cumulative downtime including undetected downtimes	Exponential with $\Lambda = 6.222E-4$	General with three moments M1=32.96 M2=2.688E+3 M3=2.530E+5
cumulative downtime risk including undetected downtimes	Exponential with $\Lambda = 6.222E-4$	General with three moments M1=5.08E-6 M2=1.94E-10 M3=3.12E-14

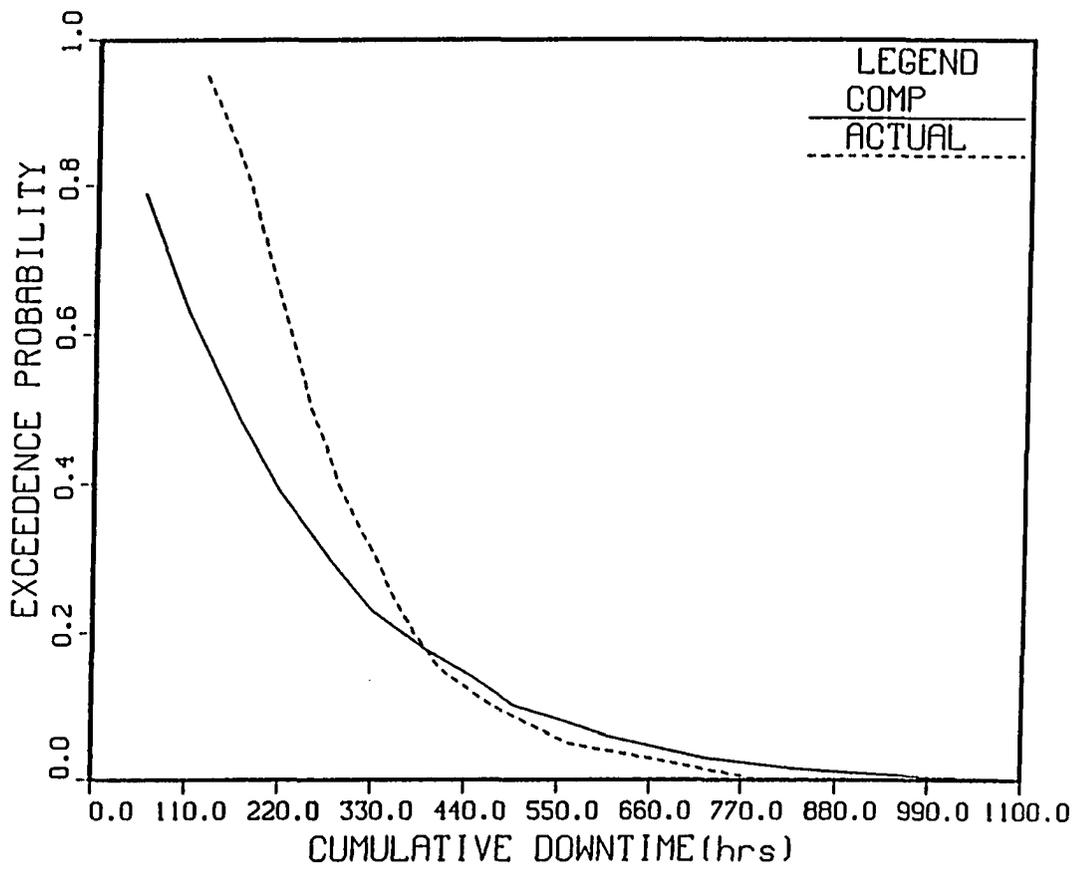


Figure 15. Comparison of compound Poisson process and actual cumulative downtime distribution (undetected = 0)

0.05 from the exact solution. The five percentile of the compound Poisson process resides at 610 hrs. This mild conservatism (about 10%) resulted from the approximations made in the compound Poisson process and it is usually of no consequence in risk based decision making processes.

#### 6.3.2 Case 2 Comparison

In this comparison, the cumulative downtime risk distribution excluding the contribution of undetected downtimes from both methods (the alternating renewal process coupled with the Monte Carlo simulation and the compound Poisson process coupled with the maximum entropy routine) were estimated. The results are graphically shown in Figure 16. Again the agreement between the two curves at the tail values are excellent. For example, the probability that the core-melt frequency exceeds  $1.0E-04$  as a result of the cumulative repair downtimes for a period of three years is 0.03 from the exact solutions and 0.04 from the approximate solutions resulted from the compound Poisson process.

#### 6.3.3 Case 3 Comparison

In this comparison, the cumulative downtime distribution including the undetected downtimes from both methods were estimated. The main difference of this evaluation with those of Case 1 and Case 2 is the use of the third moment of the downtime distribution in the compound Poisson process (see Table 8). The third moment is introduced to account of the skewness of the cumulative downtime distribution. The skewness itself incorporating the undetected downtimes in the calculations. If this evaluation would have been performed specifying only two moments for the

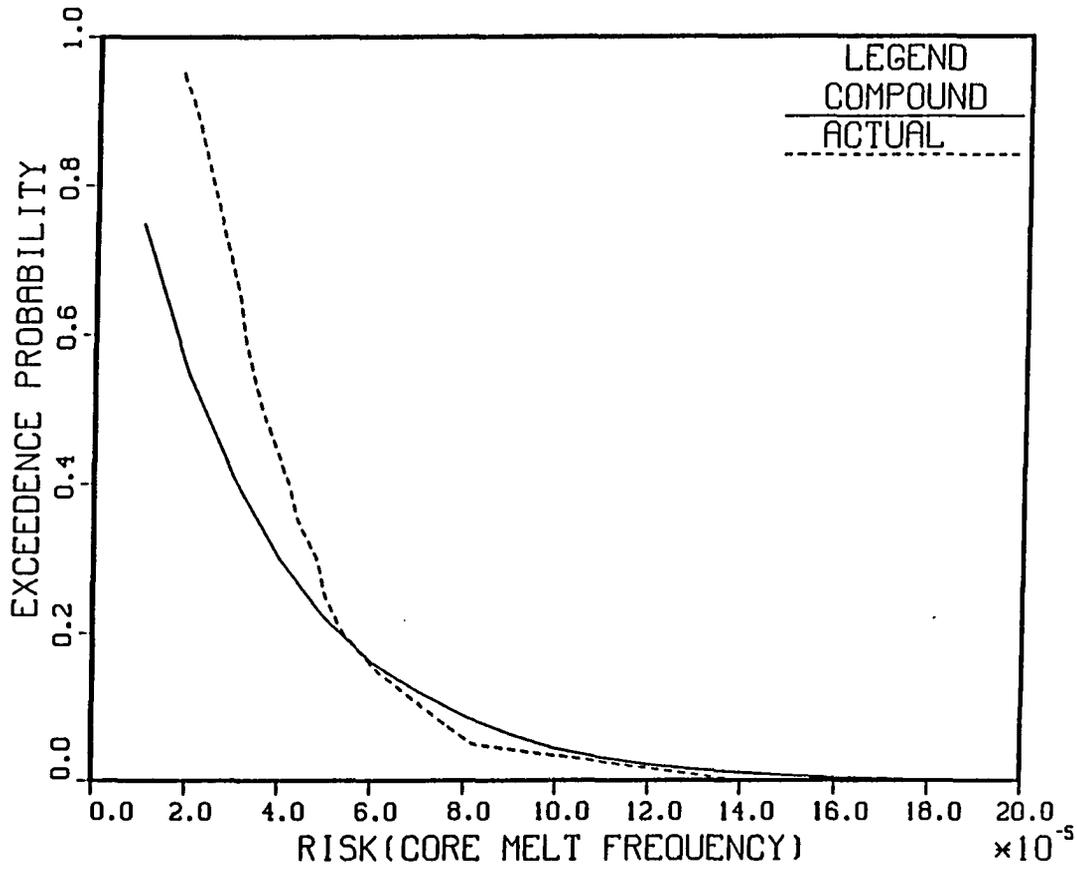


Figure 16. Comparison of compound Poisson process with the actual cumulative downtime risk distribution (undetected = 0)

downtime distribution, the resulting solution from the compound Poisson process would be a very poor approximate of the actual solution. It shall be noted that the third moment of the repair distribution is not necessary for calculating the third moment of the downtime distribution (detected plus undetected). This can be explained through the following equation:

$$M3^* = M3 + 3 T M2 + 3 T^2 M1 + T^3 \quad . \quad (124)$$

M1, M2, and M3 are the first three moments of the repair distribution, M3\* is the third moment of the downtime distribution, and finally T is the mean undetected downtime. The third moment is vital when the cumulative downtime distribution is severely skewed. In these cases, the value of T, the undetected downtime, is usually much greater than the maximum repair time. Therefore, a good approximation to the third moment of the downtime distribution can be obtained even if the third moment of the repair time is neglected (the term associated with the M3 is set equal to zero). In this trial application, neglecting M3 results in less than a one percent error in the calculated third moment. This would essentially have no effect on final results.

Figure 17 presents the results from the exact solutions and the solutions from the compound Poisson process for the cumulative downtimes of all components in the AFWS for a period of three years. Even though the agreement between the two curves (even at the tail values) is not great, it can be improved by specifying a higher number of moments for the downtime distribution. Generally the error of the approximate solution is limited to about fifty percent at the tail of the distribution.

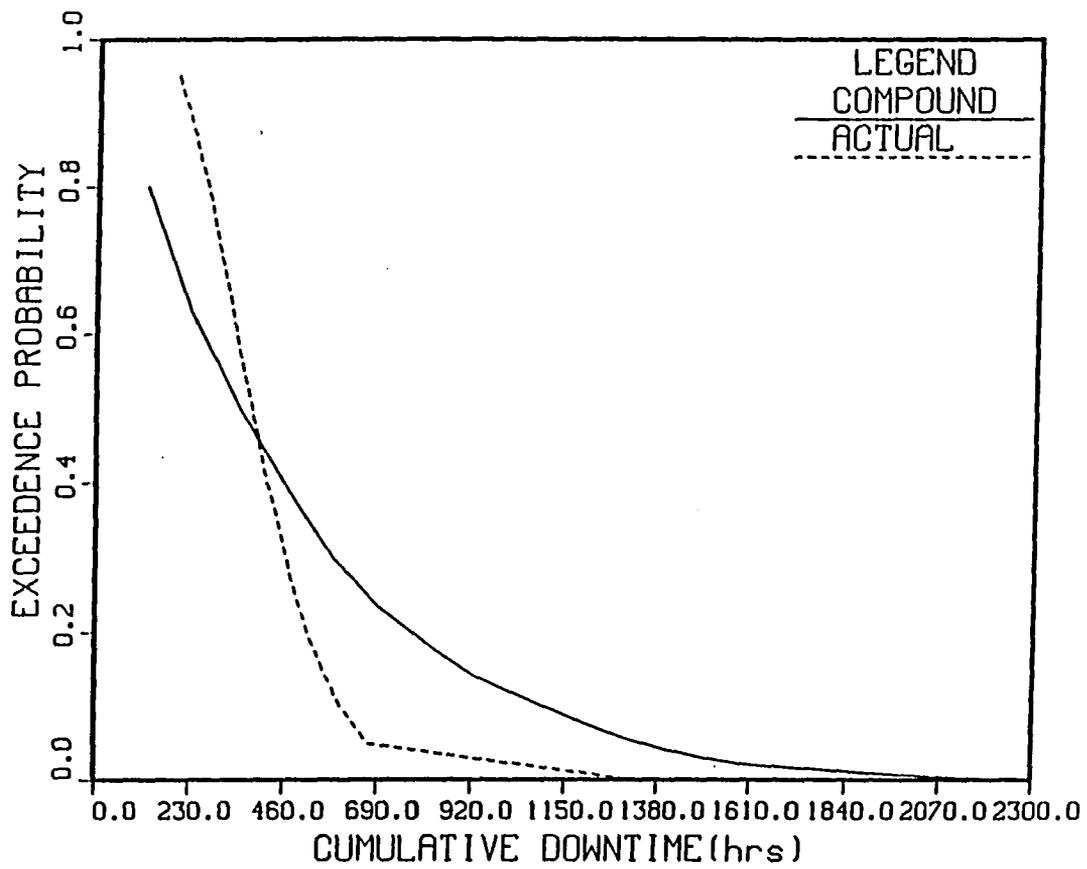


Figure 17. Comparison of compound Poisson process with actual (cumulative downtime including undetected time)

In terms of determining the allowable outage time for AFWS, this translates to a conservatism by a factor of 1.5 which is usually acceptable within the uncertainties of risk analysis.

#### 6.3.4 Case 4 Comparison

In this comparison, the cumulative downtime risk distribution including the undetected downtimes contributions were estimated from the both methods. In this case similar to the previous case the third moment of the downtime distribution is to be specified to account for the skewness of the cumulative downtime distribution.

The results of this evaluation are graphically depicted in Figure 18. The agreement of the two graphs at the tail of the distributions indicate that the compound Poisson process coupled with the maximum entropy fitting technique is a viable alternative to the exact solution especially when there is limited information about the downtime distribution. This figure shows that the probability with which the cumulative core-melt frequency in a period of three years exceeds  $2E-4$  is 0.04. The same probability from the approximate solution of compound Poisson process is about 0.055. Therefore, the approximate solutions are accurate enough for risk based decision making.

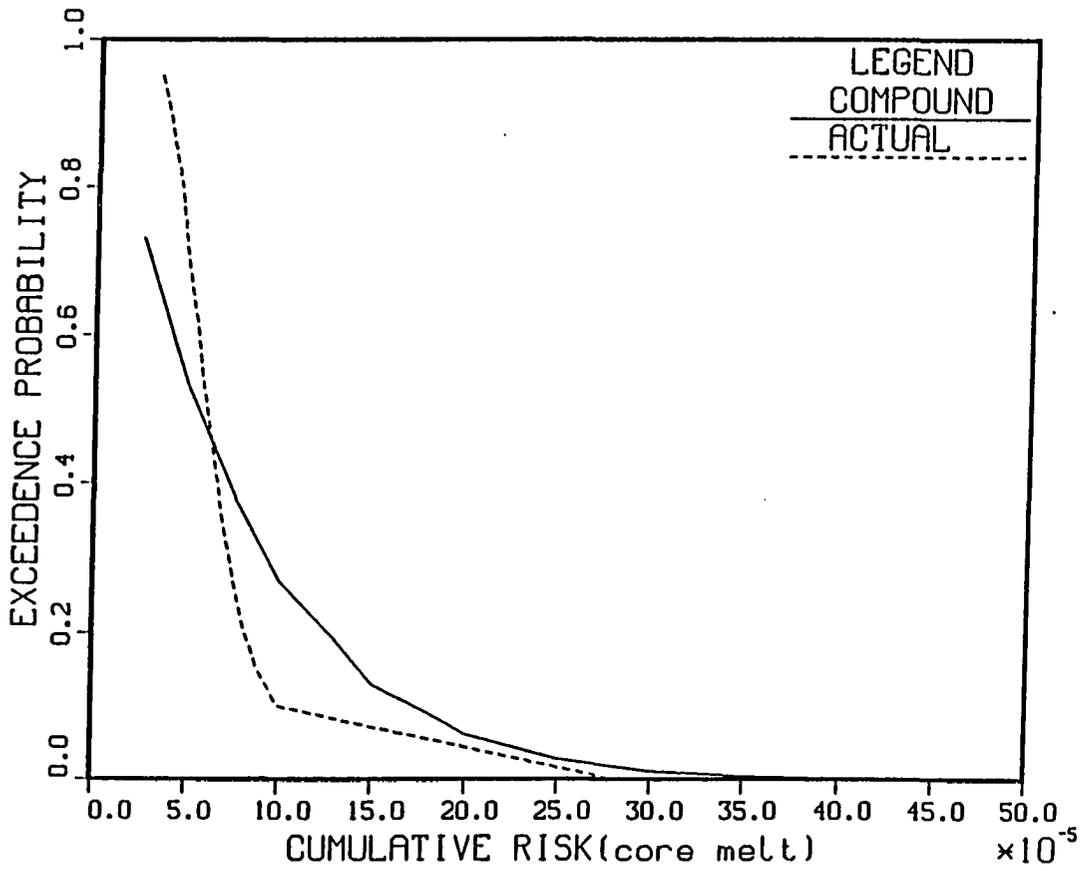


Figure 18. Comparison of compound Poisson process with actual (cumulative risk including undetected time)

## 7. CONCLUSIONS

The basic concepts and the modellings for cumulative stochastic processes have been described in this dissertation. These concepts have been further extended and formulized in the form of memory Markov processes (see Chapter 3 in general and Section 3.3 on marked Markov processes in specific).

Three approaches for modeling of cumulative stochastic processes were suggested. These are: alternating renewal process, marked Markov process, and compound Poisson process. The analytical solution techniques as well as the numerical methods including Monte Carlo simulation techniques were discussed. Two computer programs were developed to facilitate the use of these techniques.

These models then were used to address the allowable cumulative outage time for a single component and a system composed of several components. The solution techniques for obtaining the distributions of the cumulative outage time and their associated risk impact in terms of their contributions to cumulative core-melt frequency were described. An important barrier for application of these methods to the nuclear industry is recognized to be the lack of data. That is, the distributions for uptimes and downtimes for safety equipment are only partially known (e.g., only the first two moments rather than the actual or estimated distribution can be specified). A two step approach to address this problem were implemented.

The first step was to identify the impact of partial information on the accuracy of the final distributions of the cumulative downtime and cumulative downtime risk (specifically at the tail values where the interest lies). This assessment was done through sensitivity studies (Chapter 5) and the following conclusions are drawn:

- 1) The main contributor to the tail of the cumulative downtime distribution for a typical equipment in the nuclear power plants is a large number of repairs rather than few repairs with large durations. Therefore, the partial information for the uptime distribution has a large impact on the accuracy of the results, specifically at the tail values.
- 2) Out of all the uptime distributions, when the first moment is known to be equal to the variance, the exponential distribution provides the bounding results at the tail of cumulative downtime distribution. This conclusion is also applicable to the downtime distribution. Furthermore, for the downtime distribution it is shown in Section 5.2 that the impact of the variance of the downtime distribution on the tails of the cumulative downtime is insignificant as long as the change in variance is limited to a factor of two.

In the second step of the approach, the treatment of partial information was explored. The following conclusions are made:

- 1) Since the exponential uptime distribution, when applicable, (first moment and the variance are approximately the same) bounds the cumulative downtime distribution the compound

Poisson process can be used instead of the alternating renewal process. The cumulative downtime distribution, based on the moments generated by the compound Poisson process, will bound the actual distribution if the maximum entropy principle is used for specifying the distribution.

- 2) The solutions of the alternating renewal process for cumulative downtime distribution can be obtained using the partial information on moments of uptime and downtime distributions. The process involves estimating the moments of the convolutions using the moment propagation method, and specifying the actual convolution function using the maximum entropy principle. It was shown that the cumulative downtime distribution obtained in this manner would always bound the actual results at the tail values.

Furthermore, this study covered the various ways that the cumulative downtime distributions of single components can be aggregated to provide the cumulative downtime distribution of a system composed of several components. The ways to translate these cumulative downtime distributions to their associated risk contributions were also described. This information can be used by a regulator to determine a limit on the allowable cumulative downtime (or the cumulative downtime risk) for a single component as well as a system. The implications of such a decision in terms of i) its impact on plant risk, ii) its potential for being exceeded even though there is no abnormal condition (false alarm), and iii) the potential for not being exceeded when there

is an abnormal condition (miss) can be extracted from the information provided with some additional statistical analyses.

Finally, it shall be noted that this study, similar to any other research studies, has generated several areas for future work. These will be discussed in the next chapter entitled Recommendations For Future Work.

## 8. RECOMMENDATIONS FOR FUTURE WORK

On the basis of this study, there are several areas that are recognized to be important for future work on this subject. These areas are categorized in three groups: probabilistic modellings, statistical inferences, and operational evaluations.

In the probabilistic modellings, three aspects of the problem need to be investigated. These are:

- 1) The numerical as well as the analytical techniques for obtaining the solutions for the marked Markov process as it was described in Section 3.3.
- 2) The determination of the functional forms for the maximum entropy distribution based on the various types of partial information (e.g., when median and the some of the percentiles are known).
- 3) The solution techniques for obtaining the parameters of the maximum entropy distribution. It is to be noted that the Laguerre expansion method described in this dissertation for the partial information on the first few moments suffers to some extent from the changes in sign of the Laguerre polynomials. Therefore, the search for better methods of solution is to be pursued.

In the areas of the statistical analysis, there are two aspects of the problem that need to be considered for future work. These are:

- 1) Translation of past experience data into proper estimates of the uptime and downtime distributions or their characteristics. Specific techniques for testing goodness of fit and estimation methods for selection of the best distributional forms and their parameters need to be developed.
- 2) Statistical trending tests (both sequential and non-sequential) needs to be developed in order to detect an abnormal trend in cumulative downtime and their associated risk if they are monitored and tracked vs time.

Finally, in the area of operational analysis, one should investigate the various operational impacts that can be induced by imposing a limit on cumulative downtime. For example, one such impact may involve the reduction of the cumulative downtime by eliminating the scheduled preventive maintenances which may adversely impact the reliability of the safety components. This type of adverse impact can be controlled by imposing additional regulations or by monitoring other aspects of safety performance such as the reliabilities of safety equipment or systems. These types of strategies need to be identified and evaluated in order to assure that such regulatory actions do not degrade safety.

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10. APPENDIX I - LAGUERRE POLYNOMIALS AND THEIR APPLICATION  
TO ESTIMATION OF MAXIMUM ENTROPY DISTRIBUTION

Laguerre polynomials are special solutions of linear homogenous second-order differential equations related to the confluent hypergeometric differential equations. The governing differential equation is given below.

$$z \frac{d^2 w}{dz^2} + (1-z) \frac{dw}{dz} + nw = 0 \quad . \quad (10.1)$$

$w(z)$  is unique and analytic for real  $L = x$  in  $(0, \infty)$ , such that:

$$\int_0^{\infty} e^{-x} W^2(x) \quad \text{exists} \quad . \quad (10.2)$$

The orthogonality and normalization of the solutions  $L_n(x)$  is described by Equation 10.3.

$$\int_0^{\infty} e^{-x} L_n(x) L_m(x) dx = \begin{cases} 0 & (m \neq n) \\ (n!)^2 & (m=n) \end{cases} \quad (10.3)$$

The solutions of any order "n" can be written either in series form or recursion formula according to the following equations:

$$L_n(x) (-1)^n \left[ x^n - n^2 x^{n-1} + \frac{n^2(n-1)^2}{2!} x^{n-2} \pm \dots \right] \quad , \quad (10.4)$$

$$L_{n+1}(x) = (2n+1 - x) L_n(x) - n^2 L_{n-1}(x) \quad , \quad (10.5)$$

$$\frac{dL_{n+1}}{dx} = (n+1) \left[ \frac{dL_n}{dx} - L_n(x) \right] \quad . \quad (10.6)$$

Finally the generating function of Laguerre polynomials has a compact form given by Equation 10.7:

$$\sum_{n=0}^{\infty} L_n(x) \frac{S^n}{n!} = \frac{e^{-x} \frac{S}{1-S}}{1-S} \quad (0 \leq x \leq \infty) \quad . \quad (10.7)$$

The first six Laguerre polynomials can be obtained from Equation 10.4 and are provided in Table 10.1.

The Laguerre polynomial facilitate the expansion of any function for which the moments are known, that is:

$$\int f(x) x^k dx = M_k \quad . \quad (10.8)$$

In this case, if  $w(x)$  is defined by:

$$w(x) = e^x f(x) \quad , \quad (10.9)$$

then  $w(x)$  can be expanded in terms of laguerre polynomials in the following manner:

$$w(x) = \sum_i a_i L_i(x) \quad , \quad (10.10)$$

$$\int e^{-x} w(x) \cdot L_j(x) dx = L_j(M) \quad , \quad (10.11)$$

where  $L_j(M)$  is the  $j$ th Laguerre polynomial and where  $x^k$  is substituted by  $M_k$ , therefore:

$$L_j(M) = (j!)^2 a_j \quad , \quad (10.12)$$

$$a_j = \frac{L_j(M)}{(j!)^2} \quad . \quad (10.13)$$

The function  $f(x)$  then can be expanded in the form of Equation 10.14:

$$f(x) = \sum_j \left( \frac{L_j(M)}{(j!)^2} \right) L_j(x) e^{-x} \quad (10.14)$$

A conventional modification to the Laguerre expansion in Equation 10.9 is described below:

$$W(Sx) = e^{Sx} f(x) \quad (10.15)$$

is expanded through Laguerre polynomials in the following manner:

$$W(Sx) = \sum_i a_i L_i(Sx) \quad (10.16)$$

$$\int_0^{\infty} e^{-Sx} w(Sx) L_j(Sx) d(Sx) = \int f(x) L_j(Sx) d(Sx) = L_j(M^*) / (j!)^2 \quad (10.17)$$

where  $L_j(M^*)$  is the  $j$ th Laguerre polynomial and  $x^k$  is substituted by  $S^{k+1} M_k$ .

Equation 10.17 provides two useful insights. These are:

- a. The range of convergence of Laguerre expansion can be increased and the constraint defined by Equation 10.2 can be weakened by the proper choice of the value of  $S$ .
- b. The moment generating function/Laplace transform of  $f(x)$  can be obtained by analytical integration of Equation 10.16. This greatly facilitates such operations as convolutions by means of Laguerre expansion.

The Laguerre expansion was applied here to estimate a probability distribution function for which the first few moments are known. A major deficiency observed in some of these applications was the potential for of calculating negative value for the probability distribution

function at the tail values. (Probability distribution functions cannot be negative). It is obvious that if the number of moments increased the convergence would be strengthened and negative values will disappear. However, the impact of the "S" value on the negative tails was not known. This impact was studied by iterations of S values and it was shown that there is an optimum value of S for which the effect of the negative tail is minimized. The exact mathematical proof of this behavior was not investigated in this dissertation. However, an iterative process was built into the computer program to search for and identify this value of S and provide the expansion based on that S value.

The probability distribution function obtained in this manner was then translated into the form of a polynomial expansion of the powers of  $x$  (this can be done very easily by matrix operation). This polynomial expansion was then matched term by term with the Taylor expansion of the maximum entropy distribution to obtain the Lagrangian multipliers of the maximum entropy function.

Figure 10.1 depicts the actual function vs its Laguerre expansion and the estimated maximum entropy distribution. As can be seen from this illustration, the maximum entropy obtained in this manner does not necessarily conserve the original values for the moments. It also has large conservatism built into it due to the negative parts of the function estimated by the Laguerre expansion.

The next step is to use an iterative process to assure that the maximum entropy distribution conserves the moment values of the original

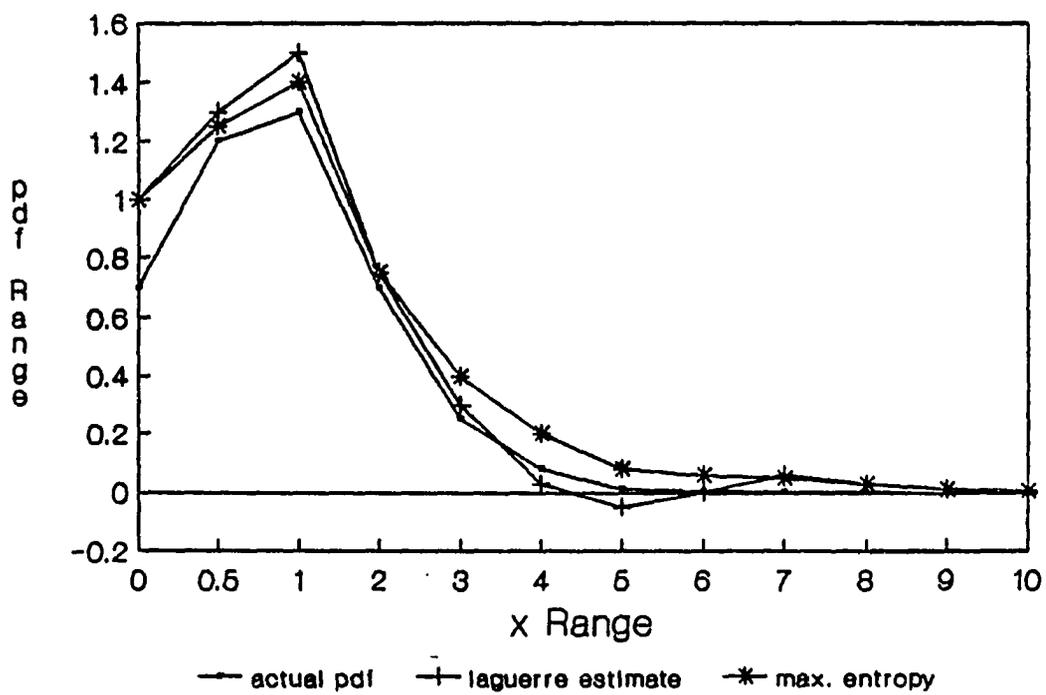


Figure 10.1 Illustration of the actual pdf vs its Laguerre and the maximum entropy estimates

distribution. This iterative process was performed using the following rationale.

$$f(x) = f^e(x) \cdot g^*(x) \quad , \quad (10.18)$$

where  $f^e(x)$  is the estimated function generated by the previously described approach,  $f(x)$  is the actual function, and  $g^*(x)$  is a perturbation function of the form:

$$g^*(x) = e^{\sum \lambda_1^* x^i} \quad , \quad (10.19)$$

the  $k$ -th moment of  $g^*(x)$  denoted as  $M_k^*$  is restricted by the following equation:

$$M_k = \int f^e(x)g^*(x) \cdot x^k dx$$

$$< \int f^e(x)x^k dx \cdot \int g^*(x)x^k dx \quad (10.20)$$

$$< M_k^e \cdot M_k^* \quad ,$$

Note that in Equation 10.19, both  $f(x)$  and  $g(x)$  are positive and  $x$  belongs to the interval  $(0, \infty)$ .  $M_k$ ,  $M_k^e$ , and  $M_k^*$  denote the  $k$ -th moment of  $f(x)$ ,  $f^e(x)$ , and  $g^*(x)$ . To proceed with the iterative process, we take  $M_k^*$  as its maximum value, that is:

$$M_k^* = M_k / M_k^e \quad . \quad (10.21)$$

Given the moments of  $g^*(x)$ , we can approximate  $g^*$  through the technique defined earlier (i.e., determining values of  $\lambda_1^*$  in Equation 10.19). We

can then define a new estimator of function  $f(x)$ , denoted as  $f^e(x)$ , that is:

$$f_n^e(x) = f^e(x) \cdot g^*(x) = \left( e^{\sum \lambda_i x^i} \right) \cdot \left( e^{\sum \lambda_i^* x^i} \right), \quad (10.22)$$

therefore, the new values for the Lamda would be

$$\lambda_i^n = \lambda_i^0 + \lambda_i^*, \quad (10.23)$$

where  $\lambda_i^n$ ,  $\lambda_i^0$  stand for the new and old value of the  $i$ -th Lamda.

To limit the computer cost, the above iterative process is continued until criteria (less than five percent errors) are satisfied for the first two moments.

11. APPENDIX II - LATIN HYPERCUBE SAMPLING AND APPROXIMATE  
SOLUTIONS TO CONVOLUTION INTEGRALS

11.1. Latin Hypercube Sampling (LHS)

Let  $x_1, x_2, \dots, x_I$  denote the  $I$  input variables. Let each  $x_i$  have a probability density function  $f_i(x)$  for  $x_i^L < x < x_i^H$ , independent of all  $x_i$ 's. Define constants  $a_{ij}$  such that  $x_i^L = a_{i0} < a_{i1} = x_i^H$  and the probability content of each interval  $(a_{ij-1}, a_{ij})$  is  $1/n$ . Let  $I_j^i$  denote the interval  $(a_{ij-1}, a_{ij})$  representing the  $j$ th interval for input variable  $i$ .

To determine the sample interval for each variable, "K", let  $U_1, U_2, \dots, U_n$  be a sequence of independent, uniform random variables. Let  $r_1$  be the rank of  $U_1$ . That is,  $r_1 - 1$  is the number of  $U_j$ 's less than  $U_1$ . The order of the sampling interval for the variable  $K$  is then based on the ranks  $r_1$  (e.g., if  $r_1 = 2$  and  $r_2 = 5$  for the variable  $K$ , the first sample is from the interval  $I_2^K$  and the second sample is from the interval  $I_5^K$ ). Repeat this process for all variables to identify all cells (combinations of intervals) that are to be sampled. Once the cells are identified, each variable interval of the cell is to be sampled to obtain specific values for the input variables.

This plan of sampling can be shown to be an improvement over simple random sampling for estimating a class of functions which includes the mean and the cumulative distribution function. The theoretical increase in precision in estimating the variance has not been found.

A Fortran 77 program and user's guide for the generation of latin hypercube and random samples for use with computer models was developed by R.L. Iman and Shortencarier, (NUREG/CR-3624, March 1984). However, the sampling procedure is simple enough that a custom made program can be written on any computer software in a short period of time.

#### 11.2. Approximate Methods for Obtaining Convolution Integrals

Let  $x_1, x_2, \dots, x_n$  denote  $n$  independent random variables with a known probability density function and  $y$  be defined as:

$$\sum_{i=1}^n x_i \quad (11.1)$$

It is tempting to approximate the probability density function of  $y$  by a normal distribution when  $n$  is large. However, in many practical cases,  $n$  is not large and furthermore  $x_i$ 's are so diversified (some large and some small) that the distribution of  $y$  is expected to deviate largely from a normal distribution. In these cases, methods such as Edgeworth expansion would provide satisfactory results only at the central parts of the  $y$  distribution and the results at the tail of the distribution are poor. Esscher's Large Deviation (ELD) technique appears to be promising for estimation of distribution tails. The central idea for the ELD is to displace the distribution of  $y$  towards the value of interest (e.g., the right tail of the distribution) that is, to induce a displaced distribution of  $y$  such that the tail values are relocated to the center of distribution. Probability determination can now be made at the central part of the shifted distribution where the normal distribution approximation applies (e.g., using Edgeworth type expansions).

Esscher's approximation can be best introduced by determining the tail probability of  $y$  when  $x_i$ 's all have the same distribution function  $F(\cdot)$ . The assumption of common distribution function is not necessary and the general ELD technique for the case when the random variables do not have common distributions is discussed by M. Mazumbar and D.P. Gaver (Reference 20).

Suppose the problem is to evaluate the probability "P" defined by:

$$P = P_r (Y > Z) = P_r \{X_1 + X_2 + \dots + X_n > Z\} \quad (11.2)$$

where  $Z$  lies on the extreme right tail of the distribution. We define, for some  $S > 0$ ,  $V(dx)$  such that

$$V(dx) = \frac{e^{SX} F(dx)}{\hat{F}(S)} \quad (11.3)$$

where

$$\hat{F}(S) = \int_{-\infty}^{\infty} e^{Sx} dF(x) \quad (11.4)$$

Let all  $X_i$  random variables be bounded. Then  $\hat{F}(S)$  is finite for  $S > 0$  and  $V(x)$  is a valid distribution function that has its mass shifted to the right of  $F(\cdot)$ .  $\hat{F}(\cdot)$  stands for function  $F$  with any given argument. Within this discussion " $\cdot$ " can replace  $dx$ . Denote the  $n$ -fold convolutions of  $F(\cdot)$  and  $V(\cdot)$  by  $F_n(\cdot)$  and  $V_n(\cdot)$ , respectively. It can be shown that

$$F_n(dx) = (dx) = [\hat{F}(s)]^n e^{-Sx} V_n(dx)$$

and therefore,

$$P = \int_Z^{\infty} F_n(dx) = [\hat{F}(s)]^n \int_Z^{\infty} e^{-Sx} V_n(dx)$$

Now choose  $S$  such that  $Z$  equals the mean of  $V_n(\cdot)$ . With this choice of  $S$  the contribution of the tails of  $V_n^*(dx)$  where the normal approximation is not valid will be reduced due to the multiplier  $e^{-Sx}$ .

To do the above we define the moment generating function of  $V_n(dx)$ , denoted by  $V_n(\xi)$ , as

$$V_n(\xi) = \frac{F_n(S + \xi)}{[\hat{F}(S)]^n}$$

Therefore the cumulant generation function of  $V_n(\cdot)$  denoted by  $K_n(\xi)$  can be defined as:

$$\begin{aligned} K_n(\xi) &= \ln V_n(\xi) = \ln(F_n(S + \xi)) - n \ln(\hat{F}(S)) \\ &= \psi_n(\xi)(S + \xi) - \psi_n(S) \end{aligned}$$

where  $\psi_n(\xi)$  is the cumulant generating function of  $F_n$ . The  $r$ th cumulant of  $V_n(\cdot)$  then is

$$K_n^{(r)}(0) = \psi_n^r(S)$$

Now put the mean of  $V_n(\cdot)$  equal to  $Z$

$$Z = K_n'(0) = \psi_n'(S)$$

and find the proper value of  $S$  and denote that by  $S_0$ . Next, replace  $V_n(dx)$  by a normal density with a mean of  $\psi_n'(S_0)$  and variance of  $\psi_n''(S_0)$ .

$V_n(dx)$  can also be better approximated by a first order Edgeworth expansion, involving the first three cumulants only (correction adjustment) that is

$$V_n(dx) = \frac{1}{[\psi'(s_0)]^{1/2}} \left[ \phi(t_0) - \frac{\gamma_1'}{6} \phi^{(3)}(t_0) \right] dx$$

where

$$t_0 = X - \frac{\psi'(s_0)}{[\psi''(s_0)]^{1/2}}$$

$$\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

$$\phi^{(3)}(t) = \frac{d^3 \phi(t)}{dt^3}$$

and

$$\gamma_1' = \frac{\psi'''(s_0)}{[\psi''(s_0)]^{3/2}}$$

Knowing the basic concept of the ELD technique for random variables with common distributions, one can generalize the results when the distributions of random variables are different. This extension of the technique mainly take advantage of the property of the moment generating function, namely translation of convolution integrals to simple multiplication (Reference 20).

This technique has proven its superiority over direct use of Edgeworth expansion.

## 12. APPENDIX III - COMPUTER PROGRAMS DEVELOPED IN THIS STUDY

There are basically four computer programs that were written for this dissertation. These are:

a) ACOT: This is a major program that calculates the cumulative downtime distribution for a single component under the general assumption of the Alternating Renewal process. In addition, this program calculates the cumulative downtime distribution for a system composed of several components for the special case of the compound Poisson process.

b) SIMUL: This program estimates the cumulative downtime distribution for a single or multi-component system using a simple Monte Carlo simulation routine.

c) SIMUL2: This program estimates the distribution of the sum of several random independent random variables each having an empirical distribution, using the Monte-Carlo sampling routine. Therefore both the cumulative downtime and the cumulative downtime risk distributions can be evaluated using this program.

d) PFM: This is a graphical program that facilitates the production of graphical outputs using the DISSPLA routine in the VAX/VMS computer system.

The user provided inputs and the program outputs are described below.

ACOT: The following inputs are required to run the ACOT program. These inputs are to be generated in a separate file called "INPUT".

## 1) NRUN:

An integer from 1 to 10, free format, indicating number of cases to be run.

## 2) T, D, DPUN, UND:

T: Total time period

D: Cumulative downtime limit due to repair

DPUN: Cumulative downtime limit due to repair plus undetected downtime

UND: Expected single outage undetected downtime. All are free formatted and real valued.

## 3) JCODE, JINDX:

JCODE: The code associated with type of model to be used; see the following inputs.

JINDX: Equal 1 for uptime information and 2 for downtime information.

IF JCODE=1 - XMU, XSIGMA: Mean & variance of normal distribution, free format, real valued

IF JCODE=2 - XL, XTN: Scale and the location parameter for an exponential distribution.

IF JCODE=3 - XNU,ALP: U and X parameters of a gamma distribution

IF JCODE=4 - S: The parameter of a Cauchy distribution

IF JCODE=5 - ALP: The parameter of a one sided stable distribution

IF JCODE=6 - P1, XMU1, XSIGMA1, XMU2, XSIGMA2: The parameters  
for the mixture of two normal distributions

IF JCODE=7 - P1, XL1, XTN1, XL2, XTN2: The parameters for the  
mixture of two exponential distribution

IF JCODE=8 - P1, XNU1, Alp2: The parameters for the mixture of  
two gamma distributions

IF JCODE=9 - XLA, NOM: The exponential parameter and the number  
of moments  
XNOM(1),...,XNOM(NOM): The values of the moments

IF JCODE=10 - NOM: Number of moments  
XMOM(1),.....,XMOM(NOM): The values of the moments

4) Repeat step 3 for JINDX=2

5) Repeat steps 3 and 4 for all the remaining cases.

The program generates a complete set of output with specific headings to show not only the final results but also intermediate results for checking purposes. The listing of this program is included at the end of this Appendix. It is important to note that the IMSLIB/LIB needs to be linked before the user attempts to execute this program.

SIMUL: The SIMUL program is written in an interactive mode. The following interactive input data are needed.

- 1) ENTER NO. OF TRAINS, SEED, NO. OF ITERATIONS, AND TOTAL TIME PERIOD
- 2) ENTER CODE FOR THE UPTIME DISTRIBUTION:
  - 1 = EXPONENTIAL
  - 2 = UNIFORM
  - 3 = EMPIRICAL

- 3) IF CODE = 1 ENTER VALUE FOR LAMDA  
IF CODE = 2 ENTER END POINTS FOR UPTIME  
IF CODE = 3 ENTER THE NAME OF THE FILE CONTAINING THE PERCENTILES
- 4) Repeat step 3 for the downtime distribution
- 5) Repeat steps 3 and 4 for all other trains.

The file containing the percentiles of the empirical distribution is free formatted and has two columns; the first is the percentiles and the second is the associated value of the variables. It is important to note that a maximum of 100 percentiles can be specified (starting with 0 and ending with 1). Note that 0 and 100 percentiles must be defined regardless of their accuracy.

The program output is an empirical distribution of cumulative downtime using Monte Carlo simulation for each percentile (0 to 1 with step increments of 0.01).

A listing of the SIMUL program is provided at the end of this appendix.

SIMUL2: This program is also written in an interactive mode similar to SIMUL. The following interactive input data is needed.

- 1) ENTER THE NAME OF THE FILE FOR EMPIRICAL DISTRIBUTION

This file shall contain the percentiles of the empirical distributions. It is free formatted and consists of two columns. The first column contains the values associated with each percentile indicated in the second column. The delimiter for separating one variable from another is "END".

2) ENTER SEED AND THE NUMBER OF VARIABLES The SEED refers to the initial seed value for the random number generator routine.

The program output is an empirical distribution representing the percentiles of the sum of the random variables. In addition to the output printed out the program automatically generates a file named "SIMOUT" containing the detailed output information. A listing of this program is given at the end of this Appendix.

PFM: This is a general purpose plot program using DISSPLA routine on VAX/VMS. The original version of this program was written by H. Connell at Brookhaven National Laboratory. This program has been slightly modified for the purpose of the graph requirements of this dissertation. This program can plot any number of plots where each plot can have up to five graphs. The program needs an input file with the assigned name "PLOTIN". An example of an input file along with the program listing is provided at the end of this appendix. After the program execution an output file named "PLOTOUT" will be generated. This file will be used by the DISSPLA routine "DISSPOP" to generate the metafile which can be used for plotting using the "TPLOT" command.

## ACOT PROGRAM

```

*c
PROGRAM CUMDON
C TEST PROGRAM FOR THE VAX/VMS
C
C DIMENSION CDT(400),CDP(400)
DIMENSION DT(20),DTU(20),DTL(20),FDF(20,20),FDT(20,20)
REAL*8 XMOM(8),XMO(8),VXMO(8)
COMMON/junk/aro,xm1o,xm2o
COMMON F(20)
C
C OPEN INPUT AND OUTPUT FILES
OPEN(UNIT=6,FILE='OUTPUT',STATUS='NEW',ERR=25)
GO TO 15
25 CONTINUE
STOP 'CANNOT OPEN OUTPUT FILE - ABORT'
15 CONTINUE
OPEN(UNIT=5,FILE='INPUT',STATUS='OLD',ERR=35,READ ONLY)
GO TO 20
35 CONTINUE
STOP 'NO INPUT FILE'
5 READ(5,*,END=2000)N
IF(N.EQ.0.OR.N.GT.10) GO TO 10000
nrun=0
NIN=NRUN+1
C T IS TOTAL TIME PERIOD,D IS TOTAL CUMULATIVE REPAIR TIME
C DPUN IS TOTAL CUMULATIVE DETECTED PLUS UNDETECTED DOWNTIME
C UND IS A SINGLE UNDETECTED DOWNTIME TAKEN AS ONE HALF TEST INTERVAL
4 read(5,*)t,d,DPUN,UND
call clear(xmom,8)
call clear(vxmo,8)
call clear(xmo,8)
call clear(dt,20)
call clear(dtu,20)
call clear(dtl,20)
call clear(fdf,400)
call clear(fdt,400)
call clear(f,20)
7 READ(5,*)JCODE,JINDX
IF(JINDX.EQ.2) GO TO 10
WRITE(6,8) JCODE,NIN
8 FORMAT(10X,'THE DOWNTIME OCCURRENCE FREQUENCY DISTRIBUTION'./,
1 25X,'JCODE = ',I2,2X,'CASE NO. = ',I2)
10 WRITE(6,8) JCODE,N
GO TO 11
8 FORMAT(10X,'THE DOWNTIME REPAIR DISTRIBUTION'./,
1 25X,'JCODE = ',I2,2X,'CASE NO. = ',I2)
11 IF(JCODE.EQ.1) GO TO 100
IF(JCODE.EQ.2) GO TO 200
IF(JCODE.EQ.3) GO TO 300
IF(JCODE.EQ.4) GO TO 400
IF(JCODE.EQ.5) GO TO 500
IF(JCODE.EQ.6) GO TO 600

```

```

      IF(JCODE.EQ.7) GO TO 700
      IF(JCODE.EQ.8) GO TO 800
      IF(JCODE.EQ.9) GO TO 900
      IF(JCODE.EQ.10) GO TO 1000
      GO TO 10000
C *** JCODE = 1 (NORMAL) ***
100 READ(5,*) XMU, XSIGMA
      WRITE(6,101) XMU, XSIGMA
101 FORMAT(25X, 'NORMAL DISTRIBUTION' ,/,
1 25X, 'MU = ', E10.2, 5X, 'SIGMA = ', E10.2, /)
      DO 102 J=1, 20
C *****
c      WRITE(6,*)J
      DD = FLOAT(J)/10.*D
      TP = T-DD-J*UND
      CALL CONVL1(TP, DD, JINDX, XMU, XSIGMA)
      IF(JINDX.EQ.2) GO TO 104
      DO 103 I=1, 20
      FDF(I, J) = F(I)
C *****TEMPORARY WRITE STATEMENTS*****
c      WRITE(6,*)FDF(I, J)
      F(I) = 0
103 CONTINUE
      GO TO 102
104 DO 105 I=1, 20
      FDT(I, J) = F(I)
C *****TEMPORARY WRITE STATEMENTS*****
c      WRITE(6,*)FDT(I, J)
      F(I) = 0
105 CONTINUE
102 CONTINUE
      IF(JINDX.EQ.1) GO TO 7
      GO TO 2000
C ***** COMBINE CONVOLUTIONS HERE *****
C *** JCODE = 2 (EXPONENTIAL) ***
200 READ(5,*) XL, XTN
      WRITE(6,201) XL, XTN
201 FORMAT(25X, 'EXPONENTIAL DISTRIBUTION' ,/,
1 25X, 'LAMBDA = ', E10.2, 5X, 'TO = ', E10.2, /)
      DO 202 J=1, 20
C*****
c      WRITE(6,*)J
      DD = FLOAT(J)/10.*D
      TP = T-DD-J*UND
      CALL CONVL2(TP, DD, JINDX, XL, XTN)
      IF(JINDX.EQ.2) GO TO 204
      DO 203 I=1, 20
      FDF(I, J) = F(I)
C *****TEMPORARY WRITE STATEMENT*****
c      WRITE(6,*)FDF(I, J)
      F(I) = 0
203 CONTINUE

```

```

      GO TO 202
204 DO 205 I=1,20
      FDT(I,J) = F(I)
C *****TEMPORARY WRITE STATEMENTS*****
c   WRITE(6,*)FDT(I,J)
      F(I) = 0
205 CONTINUE
202 CONTINUE
      IF(JINDX.EQ.1) GO TO 7
      GO TO 2000
C *** JCODE = 3 (GAMMA) ***
300 READ(5,*) XNU,ALP
      WRITE(6,301) XNU,ALP
301 FORMAT(25X,'GAMMA DISTRIBUTION',/
1 25X,'NU = ',E10.2,5X,'ALPHA = ',E10.2,/)
      DO 302 J=1,20
C*****
c   WRITE(6,*)J
      DD = FLOAT(J)/10.*D
      TP = T-DD-J*UND
      CALL CONVL3(TP,DD,JINDX,XNU,ALP)
      IF(JINDX.EQ.2) GO TO 304
      DO 303 I=1,20
          FDF(I,J) = F(I)
C *****TEMPORARY WRITES***
c   WRITE(6,*) FDF(I,J)
      F(I) = 0
303 CONTINUE
      GO TO 302
304 DO 305 I=1,20
      FDT(I,J) = F(I)
C *****TEMPORARY WRITES ***
c   WRITE(6,*) FDT(I,J)
      F(I) = 0
305 CONTINUE
302 CONTINUE
      IF(JINDX.EQ.1) GO TO 7
      GO TO 2000
C *** JCODE = 4 (CAUCHY) ***
400 READ(5,*) S
      WRITE(6,401) S
401 FORMAT(25X,'CAUCHY DISTRIBUTION',/,
1 25X,'S = ',E10.2,/)
      DO 402 J=1,20
          DD = FLOAT(J)/10.*D
          TP = T-DD-J*UND
          CALL CONVL4(TP,DD,JINDX,S)
          IF(JINDX.EQ.2) GO TO 404
          DO 403 I=1,20
              FDF(I,J) = F(I)
              F(I) = 0
403 CONTINUE

```

```

      GO TO 402
404   DO 405 I=1,20
      FDT(I,J) = F(I)
      F(I) = 0
405   CONTINUE
402   CONTINUE
      IF(JINDX.EQ.1) GO TO 7
      GO TO 2000
C *** JCODE = 5 (ONE SIDED STABLE) ***
500   READ(5,*) ALP
      WRITE(6,501) ALP
501   FORMAT(25X,'ONE SIDED STABLE DIST. OF INDEX 1/2',/,
1     25X,'ALP = ',E10.2,/)
      DO 502 J=1,20
      DD = FLOAT(J)/10.*D
      TP = T-DD-J*UND
      CALL CONVLTS(TP,DD,JINDX,ALP)
      IF(JINDX.EQ.2) GO TO 504
      DO 503 I=1,20
      FDF(I,J) = F(I)
      F(I) = 0
503   CONTINUE
      GO TO 502
504   DO 505 I=1,20
      FDT(I,J) = F(I)
      F(I) = 0
505   CONTINUE
502   CONTINUE
      IF(JINDX.EQ.1) GO TO 7
      GO TO 2000
C *** JCODE = 6 (TWO NORMAL) ***
600   READ(6,*) P1,XMU1,XSIGMA1,XMU2,XSIGMA2
      P2 = 1.-P1
      WRITE(6,601) P1,XMU1,XSIGMA1,P2,XMU2,XSIGMA2
601   FORMAT(25X,'MIXTURE OF TWO NORMAL DISTRIBUTIONS',/
1     15X,'P1 = ',F10.6,5X,'MU1 = ',E10.2,5X,'SIGMA1 = ',E10.2,/,
2     15X,'P2 = ',F10.6,5X,'MU2 = ',E10.2,5X,'SIGMA2 = ',E10.2,/)
      DO 602 J=1,20
C*****
c     WRITE(6,*)J
      DD = FLOAT(J)/10.*D
      TP = T-DD-J*UND
      CALL CONVLTS6(TP,DD,JINDX,P1,XMU1,XSIGMA1,P2,XMU2,XSIGMA2)
      IF(JINDX.EQ.2) GO TO 604
      DO 603 I=1,20
      FDF(I,J) = F(I)
C ***** TEMPORARY WRITES*****
c     WRITE(6,*) FDF(I,J)
      F(I) = 0
603   CONTINUE
      GO TO 602
604   DO 605 I=1,20

```

```

      FDT(I,J) = F(I)
C *****TEMPORARY WRITES*****
c   WRITE(6,*) FDT(I,J)
      F(I) = 0
      605 CONTINUE
      602 CONTINUE
      IF(JINDX.EQ.1) GO TO 7
      GO TO 2000
C *** JCODE = 7 (TWO EXPONENTIAL)
      700 READ(5,*) P1,XL1,XTN1,XL2,XTN2
      P2 = 1.-P1
      WRITE(6,701) P1,XL1,XTN1,P2,XL2,XTN2
      701 FORMAT(25X,'MIXTURE OF TWO EXPONENTIAL DISTRIBUTIONS',/,
1     15X,'P1 = ',F10.5,5X,'LAMBDA1 = ',E10.2,5X,'TO = ',E10.2./,
2     15X,'P2 = ',F10.5,5X,'LAMBDA2 = ',E10.2,5X,'TO = ',E10.2./)
      DO 702 J=1,20
C*****
c   WRITE(6,*) J
      DD = FLOAT(J)/10.*D
      TP = T-DD-J*UND
      CALL CONVL7(TP,DD,JINDX,P1,XL1,XTN1,P2,XL2,XTN2)
      IF(JINDX.EQ.2) GO TO 704
      DO 703 I=1,20
      FDF(I,J) = F(I)
C *****TEMPORARY WRITES*****
c   WRITE(6,*) FDF(I,J)
      F(I) = 0
      703 CONTINUE
      GO TO 702
      704 DO 705 I=1,20
      FDT(I,J) = F(I)
C *****TEMPORARY WRITES
c   WRITE(6,*) FDT(I,J)
      F(I) = 0
      705 CONTINUE
      702 CONTINUE
      IF(JINDX.EQ.1) GO TO 7
      GO TO 2000
C *** JCODE = 8 (GAMMA MIXTURE) ***
      800 READ(5,*) P1,XNU1,ALP1,XNU2,ALP2
      P2 = 1.-P1
      WRITE(6,801) P1,XNU1,ALP1,P2,XNU2,ALP2
      801 FORMAT(25X,'MIXTURE OF TWO DISTRIBUTIONS',/,
1     15X,'P1 = ',F10.5,5X,'NU1 = ',E10.2,5X,'ALPHA1 = ',E10.2./,
2     15X,'P2 = ',F10.5,5X,'NU2 = ',E10.2,5X,'ALPHA2 = ',E10.2./)
      DO 802 J=1,20
C ****
c   WRITE(6,*) J
      DD = FLOAT(J)/10.*D
      TP = T-DD-J*UND
      CALL CONVL8(TP,DD,JINDX,P1,XNU1,ALP1,P2,XNU2,ALP2)
      IF(JINDX.EQ.2) GO TO 804

```

```

      DO 803 I=1,20
        FDF(I,J) = F(I)
C ***** TEMPORARY WRITE
c      WRITE (8,*) FDF(I,J)
        F(I) = 0
      803 CONTINUE
        GO TO 802
      804 DO 805 I=1,20
        FDT(I,J) = F(I)
C***** TEMPORARY WRITES
c      WRITE(6,*) FDT(I,J)
        F(I) = 0
      805 CONTINUE
      802 CONTINUE
        IF(JINDX.EQ.1) GO TO 7
        GO TO 2000
C *** JCODE = 9 (COMPOUND POISSON PROCESS) *****
      900 READ(5,*) XLA,NOM
        IF(NOM.GT.7) GO TO 10000
        READ(5,*) (XMOM(I),I=1,NOM)
        WRITE(6,901) XLA,NOM,(I,XMOM(I),I=1,NOM)
      901 FORMAT(25X,'COMPOUND POISSON PROCESS',/,
1 25X,'DOWNTIME RATE = ',E10.2,/,
2 15X,'NUMBER OF MOMENTS = ',I2,/,
3 5(15X,I1,'MOMENT = ',E10.2,/))
c*****temporary driver to check the max.entropy****
C      FLAG=0.0
C      XMEAN=XMOM(1)+FLAG*((XMOM(2)-(XMOM(1)**2.))**0.5)
C      IF(D.LT.XMEAN)D=XMEAN
C942 XMEANN=XMOM(1)+FLAG*((XMOM(2)-(XMOM(1)**2.))**0.5)
C      DNORM=D/XMEANN
C      do 943 j=1,nom
C943 xmo(j)=xmom(j)/(xmeann**j)
C      call fit(nom,xmo,dnorm,dt,dtu,dtl,xmeann,FLAG)
C      IF(FLAG.GT.0)GO TO 942
C      go to 2050
c*****this is the end of driver*****
        xmo(1)=xmom(1)*xla*t
        DO 950 J=2,NOM
          xmo(j)=0.
          J1 = J-1
          DO 940 I=2,J
            i1=i-1
            XMO(J) = XMO(J)+COMB(j1,i1)*XMOM(J-I1)*XMO(I1)
          940 CONTINUE
          XMO(J) =(xmom(j)+XMO(J))*XLA*T
          950 CONTINUE
            flag=1.
            xmean=xmo(1)+3. *((xmo(2)-(xmo(1)**2.))**0.5)
            ddm=d
            if(d.lt.xmean)ddm=xmean
          941 xmeann=xmo(1)+flag*((xmo(2)-(xmo(1)**2.))**0.5)

```

```

          DNORM=ddm/XMeann
          DO 951 J=1,NOM
951      VXMO(J)=XMO(J)/(XMeann**J)
          CALL FIT(NOM,VXMO, DNORM,DT,DTU,DTL,XMEANN,flag)
          if(flag.gt.0.)go to 941
          IF(JCODE.EQ.9)GO TO 2051
C      go to 2050
c*****moments and maximum entropy estimates***
1000    if(jindx.eq.1)GO TO 350
        WRITE(6,351)
        GO TO 353
350    write(6,352)
352    format(5x,' the max. entropy for down rate ')
        go to 353
351    format(5x,' the max.entropy for repair distr. ')
353    read(5,*)nom
        if(nom.gt.7)stop ' too many moments'
        read(5,*)(xmom(i),i=1,nom)
        WRITE(6,355)NOM,(XmOM(I),I=1,NOM)
355    FORMAT(' NO OF MOMENTS= ',I2,/, (2X,E10.2))
        flag=1.
357    xmeann=xmom(1)+FLAG*((XMOM(2)-(XMOM(1)**2.))**0.5)
        DO 358 J=1,NOM
        XMO(J)=XMOM(J)/(XMeann**J)
        VXMO(J)=XMOM(J)
358    CONTINUE
        IF(JINDX.EQ.1)THEN
            DNORM=T/(XMeann*2)
            CALL SFIT(NOM,XMO, DNORM,DT,DTU,DTL,XMEANN,flag)
        if(flag.gt.0.)go to 357
            DO 359 I=1,20
            FDF(1,I)=1.-DT(I)
            DTU(I)=FLOAT(I)*T/20.
359    CONTINUE
            DO 310 J=1,20
            DD=FLOAT(J)/10.*D
            TP=T-DD-J*UND
            DO 311 K=1,19
            IF(TP.GT.DTU(20-K).AND.TP.LT.DTU(20-K+1))THEN
                F(J)=((FDF(1,20-K+1)-FDF(1,20-K)))
                F(J)=F(J)/(DTU(20-K+1)-DTU(20-K))
                F(J)=F(J)*(TP-DTU(20-K))+FDF(1,20-K)
            endif
311    CONTINUE
310    CONTINUE
            DO 312 K=1,20
            FDF(1,K)=F(K)
312    CONTINUE
        ELSE
            DNORM=D/Xmeann
            CALL SFIT(NOM,XMO, DNORM,DT,DTU,DTL,XMEANN,flag)
        if(flag.gt.0.)go to 357

```

```

DO 315 I=1,20
FDT(1,I)=DT(I)
DTL(I)=FLOAT(I)*D/10.
315 CONTINUE
ENDIF
DO 320 J=2,20
xmo(1)=j*xmom(1)
DO 322 I=2,NOM
SUM=0.
DO 323 K=1,I-1
SUM=SUM+COMB(I,K)*VXMO(K)*XMOM(I-K)
323 CONTINUE
XMO(I)=SUM+VXMO(I)+XMOM(I)
write(6,*)I,xmo(i)
322 CONTINUE
do 2004 i=1,nom
FDF(J,I)=1.-DT(I)
DTU(I)=FLOAT(I)*T/20.
329 CONTINUE
DO 330 JJ=1,20
DD=FLOAT(JJ)/10.*D
TP=T-DD-JJ*UND
DO 331 K=1,19
IF(TP.GT.DTU(20-K).AND.TP.LT.DTU(20-K+1))THEN
F(JJ)=(FDF(J,20-K+1)-FDF(J,20-K))
F(JJ)=F(JJ)/(DTU(20-K+1)-DTU(20-K))
F(JJ)=F(JJ)*(TP-DTU(20-K))+FDF(J,20-K)
endif
331 CONTINUE
330 CONTINUE
DO 332 K=1,20
FDF(J,K)=F(K)
332 CONTINUE
ELSE
DNORM=D/xmeann
CALL SFIT(NOM,XMO, DNORM,DT,DTU,DTL,XMEANN,flag)
DO 325 I=1,20
FDT(J,I)=1.-DT(I)
DTL(I)=FLOAT(I)*D/20.
325 CONTINUE

ENDIF
320 CONTINUE
if(jindx.eq.1)go to 7
GO TO 2000
2000 WRITE(6,2005)
2005 FORMAT(5X,'DOWNTIME',5X,'EXCEEDENCE PROBABILITY')
IF(UND.GT.0.)THEN
CALL UNDET (D,DPUN,UND,FDF,FDT,DT)
GO TO 2201
ENDIF
DO 2100 J=1,20
DT(J)=(1.-FDT(1,J))*FDF(1,J)
DO 2200 I=2,20

```

```

DT(J)=DT(J)+(FDT(I-1,J)-FDT(I,J))*FDF(I,J)
2200 CONTINUE
DD=(FLOAT(J)/10.)*DPUN
WRITE(6,*) DD, DT(J)
2100 CONTINUE
GO TO 2203
2201 DO 2202 J=1,20
DD=(FLOAT(J)/10.)*DPUN
WRITE(6,*)DD,DT(J)
2202 CONTINUE
2203 dtdl=dPUN/20.
xm3=0.
xm1=0.
xm2=0.
do 3000 j=1,20
dd=(float(20-j)+0.5)*d/10.
if(j.eq.20)then
dt(21-j)=1.-dt(21-j)
else
dt(21-j)=dt(20-j)-dt(21-j)
endif
xm1=xm1+dt(21-j)*dd
xm3=xm3+dt(21-j)*(dd**3.)
xm2=xm2+dt(21-j)*(dd**2.)
3000 continue
xm=(xm2-(xm1**2.))
write(6,3001)xm1,xm2,xm,xm3
3001 format(5x,' summary statistics moments information'
1,/,5x,' first moment',5x,' second moment',5x,' variance',
2,/, ' third moment ',/,4(5x,e10.2))
2051 nrun=nrun+1
2050 if(nrun.lt.n)go to 4
GO TO 99999
10000 WRITE(6,10001)
10001 FORMAT(1H0,'INPUT ERROR')
GO TO 99999
99999 STOP 'CUMDON'
END
SUBROUTINE CONVL1(TP, DD, JINDX, XMU, XSIGMA)
C ***** NORMAL DISTRIBUTION *****
COMMON F(20)
IF (JINDX.EQ.2)TPT=DD
IF (JINDX.EQ.1)TPT=TP
DO 10 I=1,20
XN=FLOAT(I)
ZL= (TPT-(I*XMU))/(XSIGMA*SQRT(XN))
DZL=ABS(ZL)/SQRT(2.)
IF (ZL.LE.0.)GO TO 11
F(I)=0.5+ERF(DZL)*0.5
GO TO 10
11 F(I)=0.5-ERF(DZL)*0.5
10 CONTINUE
RETURN

```

```

      END
      SUBROUTINE CONVL2(TP,DD,JINDX,XL,XTN)
C ***** EXPONENTIAL DISTRIBUTION *****
      COMMON F(20)
      IF(JINDX.EQ.2)TPT=DD
      IF(JINDX.EQ.1)TPT=TP
      SUM=1.
      DO 10 I1=1,20
      I=I1-1
      ZL= I*XTN
      D = TPT-ZL
      IF(D.LE.0.) GO TO 11
      XI=((D*XL)**I)/FACT(I)*EXP(-XL*D)
      SUM=SUM-XI
      IF(SUM.LE.0.)SUM=0.
      F(I1)=SUM
      GO TO 10
11 F(I1)=0.
10 CONTINUE
      RETURN
      END
      SUBROUTINE CONVL3(TP,DD,JINDX,XNU,ALP)
C ***** GAMMA DISTRIBUTION *****
      COMMON F(20)
      IF(JINDX.EQ.2)TPT=DD
      IF(JINDX.EQ.1)TPT=TP
      DO 10 I=1,20
      DPT=ALP*TPT
      XNUI=I*XNU
      CALL MDGAM(DPT,XNUI,PI,IERI)
      F(I)=PI
10 CONTINUE
      RETURN
      END
      SUBROUTINE CONVL4(TP,DD,JINDX,S)
C ***** CAUCHY DISTRIBUTION *****
      COMMON F(20)
      IF (JINDX.EQ.2)TPT=DD
      IF (JINDX.EQ.1)TPT=TP
      PI = 3.14
      DO 10 I=1,20
      F(I) = (PI*I*S)*(ATAN(TPT/I*S))
10 CONTINUE
      RETURN
      END
      SUBROUTINE CONVL5(TP,DD,JINDX,ALP)
C ***** ONE SIDED STABLE DISTRIBUTION OF INDEX 1/2 *****
      COMMON F(20)
      IF(JINDX.EQ.2)TPT=DD
      IF(JINDX.EQ.1)TPT=TP
      DO 10 I=1,20
      ZL= ALP/(TPT**0.5)
      DZL=ABS(ZL)/SQRT(2.)

```

```

      IF (ZL.LE.0.)GO TO 11
        F(I)=0.5+ERF(DZL)*0.5
        GO TO 10
11  F(I)=0.5-ERF(DZL)*0.5
10  CONTINUE
      RETURN
      END
      SUBROUTINE CONVL76(TP,DD,JINDX,P1,XMU1,XSIGMA1,P2,XMU2,XSIGMA2)
C  ***** MIXTURE OF TWO NORMAL DISTRIBUTIONS *****
      COMMON F(20)
        IF(JINDX.EQ.2)TPT=DD
        IF(JINDX.EQ.1)TPT=TP
      DO 10 N=1,20
        F(N)=0.
      DO 20 I=1,N+1
        I1 = I-1
        ZL=(TPT-(I1*XMU1+(N-I1)*XMU2))/(((I1*(XSIGMA1**2.))+
1  ((N-I1)*(XSIGMA2**2.))**0.5)
        IF (ZL.LE.0.)GO TO 12
        ZL=ABS(ZL)/SQRT(2.)
        ZZL=0.5+ERF(ZL)*0.5
        GO TO 11
12  ZL=ABS(ZL)/SQRT(2.)
        ZZL=0.5-ERF(ZL)*0.5
11  F(N) = F(N)+ZZL*(P1**I1)*(P2**(N-I1))*COMB(N, I1)
20  CONTINUE
10  CONTINUE
      RETURN
      END
      SUBROUTINE CONVL77(TP,DD,JINDX,P1,XL1,XTN1,P2,XL2,XTN2)
C  ***** MIXTURE OF TWO EXPONENTIAL DISTRIBUTIONS *****
      COMMON F(20)
        IF (JINDX.EQ.2)TPT=DD
        IF (JINDX.EQ.1)TPT=TP
        JJ=0
        IF(XL2.LT.XL1)GO TO 11
15  JJ=JJ+1
        X=P1
        P1=P2
        P2=X
        X=XL1
        XL1=XL2
        XL2=X
        X=XTN1
        XTN1=XTN2
        XTN2=X
        IF (JJ.EQ.2)GO TO 85
11  DL=ABS(XL1-XL2)
        DO 10 NN=1,20
          N=NN-1
          F(NN)=0.
          SUM=0.
          TCI=TPT-(N+1)*XTN2
          IF(TCI.LE.0.)GO TO 13

```

```

SUM=((P2*XL2*TCl)**N)*P2*EXP(-XL2*TCl)/FACT(N)
13 TCl=TPT-(N+1)*XTN1
IF(TCl.LE.0.)GO TO 12
SUM=SUM+((P1*XL1*TCl)**N)*P1*EXP(-XL1*TCl)/FACT(N)
12 IF(NN.EQ.1)GO TO 21
DO 20 K=2,N+1
K1=K-1
AKN=(COMB(N,K1)/(FACT(K1)*FACT(N-K1)))*((XL1/DL)*P1)**K1)*
1 ((XL2/DL)*P2)**(N-K1)
TETA1=TPT-K1*XTN1-(N-K1+1)*XTN2
TETA2=TPT-(K1+1)*XTN1-(N-K1)*XTN2
IF(TETA1.LE.0.)TETA1=0.
IF(TETA2.LE.0.)TETA2=0.
U1=DL*TETA1
U2=TETA2*DL
AKN11=AKN*EXP(-XL2*TETA1)
AKN22=AKN*EXP(-XL2*TETA2)
SUMK=0.
DO 30 M=1,N-K1+1
MM=M-1
IF(DL.EQ.0.)GO TO 50
if(u1.eq.0.)go to 123
XNM=FLOAT(N-MM)
GAM1=FACT(N-MM-1)*COMB(N-K1,MM)*P2*K1*((-1)**(N-K1-MM))
CALL MDGAM(U1,XNM,CK1,IER1)
AKN1=AKN*EXP(-XL2*TETA1)*(U1**MM)
126 SUMK=SUMK+AKN1*GAM1*CK1
123 NK1=N-K1
IF(MM.EQ.NK1)GO TO 30
if(u2.eq.0.)go to 30
GAM2=FACT(N-MM-1)*COMB(NK1-1,MM)*P1*NK1*((-1)**(NK1-1-MM))
CALL MDGAM(U2,XNM,CK2,IER2)
AKN2=AKN*EXP(-XL2*TETA2)*(U2**MM)
134 SUMK=SUMK+AKN2*GAM2*CK2
GO TO 30
50 GAM=(COMB(N-K1,MM)/(N-MM+1))*(TETA**(N+1))
SUMK=SUMK+AKN*GAM
30 CONTINUE
SUM=SUM+SUMK
20 CONTINUE
21 F(NN)=SUM+F(NN)
10 CONTINUE
IF(JJ.EQ.1)GO TO 15
85 F(1)=1.-F(1)
DO 70 NP=2,20
70 F(NP)=F(NP-1)-F(NP)
RETURN
END
SUBROUTINE CONVLTS(TP,DD,JINDX,P1,XNU1,ALP1,P2,XNU2,ALP2)
C *** MIXTURE OF TWO GAMMA DISTRIBUTIONS WITH INTEGER APPROXIMATION ***
COMMON F(20)
IF(JINDX.EQ.2)TFT=DD
IF(JINDX.EQ.1)TPT=TP

```

```

IF(ALP1.GT.ALP2)GO TO 19
X=ALP1
ALP1=ALP2
ALP2=X
X=XNU1
XNU1=XNU2
XNU2=X
X=P1
P1=P2
P2=X
19 NU1=INT(XNU1)
   NU2=INT(XNU2)
   IF(NU1.LE.0.)NU1=1
   IF(NU2.LE.0.)NU2=1
   DALP=ALP1-ALP2
   A1T=ALP1*TPT
   A2T=ALP2*TPT
   DTT=DALP*TPT
   DUL=DALP/ALP1
   DO 10 N=1,20
   SUM=0.
   NN2=N*NU2
   NN1=N*NU1
   XNN2=FLOAT(NN2)
   XNN1=FLOAT(NN1)
   CALL MDGAM(A2T,XNN2,P5,IER2)
   CALL MDGAM(A1T,XNN1,P6,IER1)
   SUM=SUM+P6+P5
   N1=N-1
   IF(N1.EQ.0) GO TO 40
   DO 20 K=1,N1
   M1K=K*NU1-1
   M2K=(N-K)*NU2-1
   NK=(N-K)
   SK=(P1**K)*(P2**NK)*COMB(N,K)*ALP1*ALP2
   SK=SK*EXP(-A2T)/(FACT(M1K)*FACT(M2K))
   SUMM=0.
   IF(M1K.GT.0.)GO TO 21
   SUMM=SUMM+((A2T**M2K)/DALP)*(1.-EXP(-DTT))
   GO TO 22
21 M1K1=M1K+1
   XM1K1=FLOAT(M1K1)
   CALL MDGAM(DTT,XM1K1,P3,IER3)
   P3=P3/(DALP*(DUL**M1K))
   SUMM=SUMM+P3*(A2T**M2K)*M1K/FACT(M1K)
22 IF(M2K.EQ.0) GO TO 31
   DO 30 M=1,M2K
   MKM=M1K+M+1
   MKMM=MKM-1
   M2M=M2K-M
   XMKM=FLOAT(MKM)
   CALL MDGAM(DTT,XMKM,P,IER)
   P=P*(A2T**M2K)/((DTT**M)*(DUL**M1K)*DALP*MKMM)

```

```

          SUMM=SUMM+P*COMB(M2K,M)
30      CONTINUE
31      SUM=SUM+SK*SUMM
20      CONTINUE
C*****
      WRITE(6,*) N,F
40      F(N)=SUM
10      CONTINUE
      RETURN
      END
      FUNCTION FACT(N)
      IF(N.EQ.0) GO TO 10
      FACT=FACT(N)
      GO TO 99
10      FACT=1.
99      RETURN
      END
      FUNCTION COMB(N,M)
      IF(N.LT.M)THEN
          NX=M
          Mx=N
      else
          nx=n
          mx=m
      ENDIF
      IF(N.EQ.0) GO TO 10
      IF(M.EQ.0) GO TO 10
      COMB=fact(Nx)/(fact(mx)*fact(nx-Mx))
      GO TO 99
10      COMB=1
99      RETURN
      END
C THIS PROGRAM ATTEMPTS TO FIT THE MAXIMUM ENTROPY DISTRIBUTION
C TO A SET OF MOMENTS.THE MAXIMUM NO OF MOMENTS IS ESVEN
C N: IS THE NO OF MOMENTS,DM IS A VECTOR,RLAMD IS THE VECTOR
C OF LANGRANGEAN MULTIPLIERS.
      subroutine maxen(n,DM,rlamd,IFLAG)
      REAL*8 WL(8,8)/1.,1.,2.,8.,24.,120.,720.,5040.,0.,-1.,-4.,
1-18.,-96.,-600.,-4320.,-35280.,2*0.,1.,9.,72.,600.,
25400.,52920.,3*0.,-1.,-18.,-200.,-2400.,-29400.,4*0.,
31.,25.,450.,7350.,5*0.,-1.,-36.,-882.,6*0.,1.,49.,
47*0.,-1./
      REAL*8 WD(8,8),WJ(8,8),WP(8,8),WF(8,8),WJJ(8,8)
      REAL*8 TM(8),DM(8)
      real*8 rlamd(8)
      REAL*8 SUM,F
      IF(N.GT.7)STOP ' TOO MANY MOMENTS'
      CALL DCLEAR(WJJ,84)
      CALL DCLEAR(WF,64)
      CALL DCLEAR(WJ,64)
      CALL DCLEAR(TM,8)
      CALL DCLEAR(WP,64)
      CALL DCLEAR(RLAMD,8)

```

```

      N=N+1
      WF(1,1)=1.
      TM(1)=1.
      DO 10 I=2,N
      WF(I,I)=1./(FACT(I-1)**2.)
10     TM(I)=DM(I-1)
      CALL PRODM(WF,WL,WJJ,N,N)
      CALL TRANS(wj,wjj,N,N)
      CALL PRODM(WJ,WL,WP,N,N)
      DO 30 I=1,N
      F=0.
      DO 40 J=1,N
40     F=F+TM(J)*WP(J,I)
      RLAMD(I)=F
30     continue
      if(rlamd(1).le.0.) then
      j1=j-1
      DO 60 II=3,JJ
      I=II-1
      I1=I-1
      SUM=SUM+RLAMD(J-I+2)*TM(I)*float(j-i+1)
60     CONTINUE
      sum=sum/float(j)
      RLAMD(JJ)=(TM(JJ)-SUM)/TM(1)
50     CONTINUE
      n=n-1
      RLAMD(2)=RLAMD(2)-1.
c*****temporay write statement*****
C     write(6,*) (TM(i),i=1,n+1)
c     write(6,*) n+1,rlamd(n+1),tm(n+1)
      return
      END
C
C THIS SUBROUTINE IS USED FOR CALCULATING THE
C PRODUCT OF TWO MATRICES.
      SUBROUTINE PRODM(W1,W2,W3,P,Q)
      REAL*8 W1(8,8),W2(8,8),W3(8,8)
      DO 10 I=1,P
      DO 20 K=1,Q
      W3(I,K)=0.
      DO 30 J=1,Q
30     W3(I,K)=W3(I,K)+W1(I,J)*W2(J,K)
      continue
20     CONTINUE
10     CONTINUE
      RETURN
      END
C
C THIS SUBROUTINE GETS THE TRANSPOSE OF A MATRIX WT WITH P
C ROWS AND Q COLUMNS.
      SUBROUTINE TRANS(WT,WO,P,Q)
      INTEGER P,Q
      REAL*8 WT(8,8),WO(8,8)
      DO 10 I=1,P

```

```

      DO 20 K=1,Q
20     WT(K,I)=WO(I,K)
10     CONTINUE
      RETURN
      END
C THIS SUBROUTINE CLEARES A VECTOR OR A MATRIX
C OF SIZE N, FOR A MATRIX N=P*Q
C
      SUBROUTINE CLEAR(VEC,N)
      REAL VEC(N)
      DO 2 I=1,N
      VEC(I)=0.0
2     CONTINUE
      RETURN
      END
C THIS SUBROUTINE CLEARS A VECTOR OR A MATRIX
C OF SIZE N, FOR A MATRIX N=P*Q
C
      SUBROUTINE DCLEAR(DVEC,N)
      REAL*8 DVEC(N)
      DO 2 I=1,N
      DVEC(I)=0.0
2     CONTINUE
      RETURN
      END
C
C THIS SUBROUTINE ATTEMPTS TO FIT THE CUMULATIVE DOWNTIME
C DISTRIBUTION USING COMPOUND POISSON PROCESS WITH THE
C HELP OF MAXIMUM ENTROPY PRINCIPLE AND LAGUERRE EXPANSION
C NO. OF MOMENTS GENERATED FROM COMPOUND POISSON, XMO IS A
C VECTOR CONTAINING THE MOMENTS AND RLAMDA IS A VECTOR
C CONTAINING THE LANGRAGEAN LAMDAS. DT THEN WOULD BE A
C CONTAINING THE EXCEEDENCE PROBABILITIES.
      SUBROUTINE FIT(NOM,XMO,D,DT,DTU,DTL,XNORMA,flag)
      DIMENSION DT(20),DTU(20),DTL(20),FDF(20,20),FDT(20,20)
      REAL*8 XMOM(8),XMO(8)
      REAL*8 RLAMDA(8)
      real*8 dd,ddd,dtdd
      common/junk/aro,xm1o,xm2o
      real*8 same,samen,sameo,xm1,xm2,earg,earg2
      jflag=0
      IFLAG=0
      iiflag=0
      CALL MAXEN(NOM,XMO,RLAMDA,IFLAG)
      call clear(DT,20)
      CALL CLEAR(DTL,20)
      nom=nom+1
52     if(iiflag.eq.2)go to 51
      dtd=d/10.
      dtdd=dtd/40.
      xm1=0.
      xm2=0.
      do 10 j=1,20

```

```

dd=float(j-1)/10.*d
earg=rlamda(1)
if(j.eq.1)then
    sameo=dexp(earg)
    same=0.
    go to 41
else
do 20 i=2,nom
i1=i-1
20 earg=earg+(dd**i1)*rlamda(i)
if(earg.gt.72.0)earg=72.0
sameo=dexp(earg)
same=0.
endif
41 do 40 k=1,40
earg2=rlamda(1)
ddd=dd+dtdd*float(k)
do 42 i=2,nom
i1=i-1
42 earg2=earg2+(ddd**i1)*rlamda(i)
if(earg2.gt.72)earg2=72
samem=dexp(earg2)
same=same+(samem+sameo)*0.5*dtdd
sameo=samem
xm1=xm1+(samem*ddd+sameo*(ddd-dtdd))*0.5*dtdd
xm2=xm2+(samem*(ddd**2.)+sameo*((ddd-dtdd)**2.))*0.5*dtdd
40 continue
dt(j)=same
if(j.gt.1)dt(j)=dt(j)+dt(j-1)
10 continue
xx1=abs(xm1-xmo(1))/xmo(1)
xx2=abs(xm2-xmo(2))/xmo(2)
xx=(dt(20)-dt(19))
xxa=abs(dt(20)-1.)
if(((xx.gt.0.01).OR.(IFLAG.EQ.1).or.(xx1.gt.0.5)
1.or.(xx2.gt.2.0).or.(xxa.gt.0.07)).and.(iiflag.eq.0))then
c*****temporary write statement*****
write(6,*)dt(20),XX,IFLAG,XX1,XX2
    flag=flag+1.0
    write(6,*)flag
    if(flag.gt.10.)stop' no convergence in 10 iterations'
    go to 39
else
    if(iiflag.eq.1)go to 105
    write(6,*)dt(20),xx,iiflag,xx1,xx2
    jflag=int((flag-int(flag))*10.)+1
    if(jflag.gt.6)go to 105
    if(jflag.eq.1)then
        aro=xxa
        xm1o=xx1
        xm2o=xx2
        flag=flag+0.1
        go to 39
    else

```

```

        arn=xxa
        xmin=xx1
        xm2n=xx2
    endif
    if((arn.lt.aro).and.(xmin.lt.xm1o).and.(xm2n.lt.xm2o))then
        flag=flag+0.1
        aro=arn
        xm1o=xmin
        xm2o=xm2n
        go to 39
    endif
105    flag=0.
    endif
c*****temporary write statement*****
    do 102 i=1,nom
102    xmom(i)=xmo(i)*(xnorma**i)
        write(6,100)
100    format(2x,' the actual moments')
        write(6,*) (xmom(k),k=1,nom)
        write(6,30)
30    format(2x,' the values of lamda for max.enthgropy fit')
        write(6,*) (rlamda(k),k=1,nom)
        xm1=xm1*xnorma
        xm2=xm2*(xnorma**2.)
26    write(6,26)dt(20),xm1,xm2
        format(/,5x,' total area= 'e10.4,' m1,m2',2(4x,e10.4))
        a0=((xmom(1)/xm1)**0.667)*((xm2/xmom(2))**0.834)/(dt(20)**1.17)
        a1=((xm2/xmom(2))**0.5)/(dt(20)**0.5)
        rlamda(1)=rlamda(1)+alog(a0)
        do 50 j=2,nom
50    rlamda(j)=(a1**float(j-1))*rlamda(j)
            continue
            iflag=1+iflag
            go to 52
51    write(6,32)
32    format(4x,'cumulative downtime exceedence probability')
        do 31 j=1,20
            dt(j)=dt(j)/dt(20)
            dt(j)=1.-dt(j)
            dd=float(j)*XNORMA*d/10.
            write(6,33)dd,dt(j)
33    format(4x,e10.2,4x,e10.2)
31    continue
39    nom=nom-1
        return
    end
    FUNCTION FAC(N)
    FAC=1.
    DO 10 I=1,N
10    FAC=FAC*I
    RETURN

```

```

END
SUBROUTINE SFIT(NOM,XMO,DAC,DT,DTU,DTL,XNORMA,flag)
DIMENSION DT(20),DTU(20),DTL(20),FDF(20,20),FDT(20,20)
REAL*8 XMOM(8),XMO(8)
REAL*8 RLAMDA(8)
real*8 dd,ddd,dtdd
common/junk/aro,xm1o,xm2o
real*8 same,samen,sameo,xm1,xm2,earg,earg2
IDFLAG=0
jflag=0
IFLAG=0
iiflag=0
CALL MAXEN(NOM,XMO,RLAMDA,IFLAG)
call clear(DT,20)
CALL CLEAR(DTL,20)
D=XMO(1)+2.5*((XMO(2)-XMO(1)**2.)*0.5)
nom=nom+1
52  if(iiflag.eq.2)go to 51
200 dtd=d/10.
    dtdd=dtdd/40.
    xm1=0.
    xm2=0.
    do 10 j=1,20
    dd=float(j-1)/10.*d
    earg=rlamda(1)
    if(j.eq.1)then
        sameo=dexp(earg)
        same=0.
        go to 41
    else
    do 20 i=2,nom
    i1=i-1
    20  earg=earg+(dd**i1)*rlamda(i)
        if(earg.gt.72.0)earg=72.0
        sameo=dexp(earg)
        same=0.
    endif
    41  do 40 k=1,40
        earg2=rlamda(1)
        ddd=dd+dtdd*float(k)
        do 42 i=2,nom
        i1=i-1
    42  earg2=earg2+(ddd**i1)*rlamda(i)
        if(earg2.gt.72)earg2=72
        samen=dexp(earg2)
        same=same+(samen+sameo)*0.5*dtdd
        sameo=samen
        xm1=xm1+(samen*ddd+sameo*(ddd-dtdd))*0.5*dtdd
        xm2=xm2+(samen*(ddd**2.)+sameo*((ddd-dtdd)**2.))*0.5*dtdd
    40  continue
        dt(j)=same
        if(j.gt.1)dt(j)=dt(j)+dt(j-1)
    10  continue

```

```

IF(IDFLAG.EQ.1)GO TO 201
XMO=DT(20)
xx1=abs(xm1-xmo(1))/xmo(1)
xx2=abs(xm2-xmo(2))/xmo(2)
xx=(dt(20)-dt(19))
xxa=abs(dt(20)-1.)
if(((xx.gt.0.05).OR.(IFLAG.EQ.1)).or.(xx1.gt.0.5)
1.or.(xx2.gt.2.0).or.(xxa.gt.0.07)).and.(iiflag.eq.0))then
c*****temporary write statement*****
write(6,*)dt(20),XX,IFLAG,XX1,XX2
flag=flag+1.0
write(6,*)flag
if(flag.gt.10.)stop' no convergence in 10 iterations'
go to 39
else
if(iiflag.eq.1)go to 105
xmn=xx1
xm2n=xx2
endif
if((arn.lt.aro).and.(xmn.lt.xm1o).and.(xm2n.lt.xm2o))then
flag=flag+0.1
aro=arn
xm1o=xmn
xm2o=xm2n
go to 39
endif
105 flag=0.
endif
c*****temporary write statement*****
do 102 i=1,nom
102 xmom(i)=xmo(i)*(xnorma**i)
xm1=xm1*xnorma
xm2=xm2*(xnorma**2.)
write(6,26)dt(20),xm1,xm2
26 format(/,5x,' total area= 'e10.4,' m1,m2',2(4x,e10.4))
a0=((xmom(1)/xm1)**0.667)*((xm2/xmom(2))**0.834)/(dt(20)**1.17)
rlamda(1)=rlamda(1)+alog(a0)
do 50 j=2,nom
rlamda(j)=(a1**float(j-1))*rlamda(j)
50 continue
iiflag=1+iiflag
go to 52
51 IDFLAG=IDFLAG+1
D=DAC
GO TO 200
201 DO 31 J=1,20
DT(J)=1.-(DT(J)/XMO)
31 CONTINUE
39 nom=nom-1
return
end

```

```

C THIS SUBROUTINE TAKES CARE OF UNDETECTED DOWNTIME
C BY KEEPING TRACK ON NUMBER OF FAILURES
SUBROUTINE UNDET (D,DPUN,UND,FDF,FDT,DT)
DIMENSION FDF(20,20),FDT(20,20),DT(20)
DIMENSION CDT(400),CDP(400)
DO 10 I=1,20
DO 20 J=1,20
IF(I.EQ.1)THEN
    CDP(J)=(1.-FDT(1,J))*FDF(1,J)
    CDT(J)=UND+FLOAT(J)*D/10.
    GO TO 20
ENDIF
IJ=(I-1)*20+J
CDP(IJ)=(FDT(I-1,J)-FDT(I,J))*FDF(I,J)
CDT(IJ)=UND*I+FLOAT(J)*D/10.
20 CONTINUE
10 CONTINUE
DO 40 I=1,20
DPRIME=DPUN*FLOAT(I)/10.
DT(I)=0.
DO 50 J=1,20
KI=(J-1)*20+1
KF=J*20
DO 60 K=KI,KF
IF(CDT(K).GE.DPRIME)THEN
    DT(I)=DT(I)+CDP(K)
    GO TO 50
ENDIF
60 CONTINUE
50 CONTINUE
40 CONTINUE
RETURN
END

```

## SIMUL PROGRAM

```

*c
PROGRAM SIMUL
COMMON/ALI/UP(101),DP(101)
DIMENSION VEC(1000),D(1000,5)
DIMENSION JCODE(5),ICODE(5)
WRITE (6,2)
2 1 TOTAL TIME PERIOD')
  FORMAT( ' ENTER NO OF TRAINS,SEED,NO OF ITERATIONS,AND
  READ(5,*)NT,KSEED,NITER,TVALUE
  DO 3 I=1,NT
  WRITE(6,4)
4 1 1 FOR EXPONENTIAL 2 FOR UNIFORM 3 FOR EMPERICAL ')
  FORMAT( ' ENTER THE CODE FOR THE UPTIME DISTRIBUTION
  READ(5,*)JCODE(I)
  IF(JCODE(I).EQ.3)THEN
  WRITE(6,6)
6 2 THE PERCENTILES OF UPTIME DISTRIBUTION 0 TO 1 ')
  FORMAT( ' ENTER THE NAME FOR THE FILE CONTAINING THE
  READ(5,7)UPFILE
  FORMAT(A)
  OPEN(FILE=UPFILE,UNIT=1,STATUS='OLD',ERR=9,READ ONLY)
  CONTINUE
  STOP ' NO INPUT FILE FOR UPTIME '
8  CALL EMPDU
  ENDF
  WRITE(6,10)
10 1 1 FOR EXPONENTIAL 2 FOR UNIFORM 3 FOR EMPERICAL ')
  FORMAT( ' ENTER THE CODE FOR THE DOWNTIME DISTRIBUTION
  READ(5,*)ICODE(I)
  IF(ICODE(I).EQ.3)THEN
  WRITE(6,11)
11 2 PERCENTILES OF DOWNTIME DISTRIBUTION 0 TO 1 ')
  FORMAT( ' ENTER THE NAME FOR THE FILE CONTAINING THE
  READ(5,12)DNFILE
  FORMAT(A)
  OPEN(FILE=DNFILE,UNIT=2,STATUS='OLD',ERR=13,READ ONLY)
  GO TO 14
13 CONTINUE
  STOP ' NO INPUT FILE FOR THE DOWNTIME '
14 CALL EMPDD
  ENDF
  IF((JCODE(I).EQ.1).AND.(ICODE(I).EQ.1))GO TO 20
  IF((JCODE(I).EQ.1).AND.(ICODE(I).EQ.2))GO TO 30
  IF((JCODE(I).EQ.1).AND.(ICODE(I).EQ.3))GO TO 40
  IF((JCODE(I).EQ.2).AND.(ICODE(I).EQ.1))GO TO 50
  IF((JCODE(I).EQ.2).AND.(ICODE(I).EQ.2))GO TO 60
  IF((JCODE(I).EQ.2).AND.(ICODE(I).EQ.3))GO TO 70
  IF((JCODE(I).EQ.3).AND.(ICODE(I).EQ.1))GO TO 80
  IF((JCODE(I).EQ.3).AND.(ICODE(I).EQ.2))GO TO 90
  IF((JCODE(I).EQ.3).AND.(ICODE(I).EQ.3))GO TO 100
  STOP 'VALUE OF THE JCODE OUT OF RANGE '
20 WRITE(6,21)
21 2 DISTRIBUTIONS ')
  FORMAT( ' ENTER VALUES OF LAMDA FOR UPTIME AND DOWNTIME
  READ(5,*)ULAM,DLAM

```

```

NIT=0
22  NIT=NIT+1
    IF(NIT.GT.NITER)GO TO 3
    UTIME=0.
    DTIME=0.
25  WU=-ALOG(1.-RAN(KSEED))/ULAM
    WD=-ALOG(1.-RAN(KSEED))/DLAM
    UTIME=UTIME+WU
    IF(UTIME.GT.TVALUE)GO TO 23
    DTIME=DTIME+WD
    T=UTIME+DTIME
    IF(T.GT.TVALUE)GO TO 24
    GO TO 25
23  D(NIT,I)=DTIME
    GO TO 22
24  D(NIT,I)=DTIME+TVALUE-T
    GO TO 22
30  WRITE(6,31)
31  FORMAT( ' ENTER VALUE OF LAMDA FOR UPTIME AND THE ENDPOINTS OF
3  DOWNTIME DISTRIBUTIONS ')
    READ(5,*)ULAM,DINIT,DFINAL
    NIT=0
32  NIT=NIT+1
    IF(NIT.GT.NITER)GO TO 3
    UTIME=0.
    DTIME=0.
35  WU=-ALOG(1.-RAN(KSEED))/ULAM
    WD=RAN(KSEED)*(DFINAL-DINIT)+DINIT
    UTIME=UTIME+WU
    IF(UTIME.GT.TVALUE)GO TO 33
    DTIME=DTIME+WD
    T=UTIME+DTIME
    IF(T.GT.TVALUE)GO TO 34
    GO TO 35
33  D(NIT,I)=DTIME
    GO TO 32
34  D(NIT,I)=DTIME+TVALUE-T
    GO TO 32
40  WRITE(6,41)
41  FORMAT( ' ENTER VALUE OF LAMDA FOR UPTIME AND NOTHING FOR
3  DOWNTIME DISTRIBUTIONS ')
    READ(5,*)ULAM
    NIT=0
42  NIT=NIT+1
    IF(NIT.GT.NITER)GO TO 3
    UTIME=0.
    DTIME=0.
45  WU=-ALOG(1.-RAN(KSEED))/ULAM
    JP=INT(RAN(KSEED)*100)+1
    WD=DP(JP)
    UTIME=UTIME+WU
    IF(UTIME.GT.TVALUE)GO TO 43
    DTIME=DTIME+WD
    T=UTIME+DTIME

```

```

IF(T.GT.TVALUE)GO TO 44
GO TO 45
43 D(NIT,I)=DTIME
GO TO 42
44 D(NIT,I)=DTIME+TVALUE-T
GO TO 42
50 WRITE(6,51)
51 3 FORMAT( ' ENTER END POINTS FOR UPTIME AND LAMDA FOR
DOWNTIME DISTRIBUTIONS ')
READ(5,*)UINIT,UFINAL,DLAM
NIT=0
52 NIT=NIT+1
IF(NIT.GT.NITER)GO TO 3
UTIME=0.
DTIME=0.
55 WU=RAN(KSEED)*(UFINAL-UINIT)+UINIT
WD=-ALOG(1.-RAN(KSEED))/DLAM
UTIME=UTIME+WU
IF(UTIME.GT.TVALUE)GO TO 53
DTIME=DTIME+WD
T=UTIME+DTIME
IF(T.GT.TVALUE)GO TO 54
GO TO 55
53 D(NIT,I)=DTIME
GO TO 52
54 D(NIT,I)=DTIME+TVALUE-T
GO TO 52
60 WRITE(6,61)
61 3 FORMAT( ' ENTER END POINTS FOR BOTH UPTIME AND
DOWNTIME DISTRIBUTIONS ')
READ(5,*)UINIT,UFINAL,DINIT,DFINAL
NIT=0
62 NIT=NIT+1
IF(NIT.GT.NITER)GO TO 3
UTIME=0.
DTIME=0.
65 WU=RAN(KSEED)*(UFINAL-UINIT)+UINIT
WD=RAN(KSEED)*(DFINAL-DINIT)+DINIT
UTIME=UTIME+WU
IF(UTIME.GT.TVALUE)GO TO 63
DTIME=DTIME+WD
T=UTIME+DTIME
IF(T.GT.TVALUE)GO TO 64
GO TO 65
63 D(NIT,I)=DTIME
GO TO 62
64 D(NIT,I)=DTIME+TVALUE-T
GO TO 62
70 WRITE(6,71)
71 3 FORMAT( ' ENTER END POINTS FOR UPTIME AND NOTHING FOR
DOWNTIME DISTRIBUTIONS ')
READ(5,*)UINIT,UFINAL
NIT=0
72 NIT=NIT+1

```

```

      IF(NIT.GT.NITER)GO TO 3
      UTIME=0.
      DTIME=0.
75     WU=RAN(KSEED)*(UFINAL-UINIT)+UINIT
      JP=RAN(KSEED)*100+1
      WD=DP(JP)
      UTIME=UTIME+WU
      IF(UTIME.GT.TVALUE)GO TO 73
      DTIME=DTIME+WD
      T=UTIME+DTIME
      IF(T.GT.TVALUE)GO TO 74
      GO TO 75
73     D(NIT,I)=DTIME
      GO TO 72
74     D(NIT,I)=DTIME+TVALUE-T
      GO TO 72
80     WRITE(6,81)
81     FORMAT( ' ENTER NOTHING FOR UPTIME AND THE VALUE FOR LAMDA FOR
3     DOWNTIME DISTRIBUTIONS ')
      READ(5,*)DLAM
      NIT=0
82     NIT=NIT+1
      IF(NIT.GT.NITER)GO TO 3
      UTIME=0.
      DTIME=0.
85     JP=INT(RAN(KSEED)*100.)+1
      WU=UP(JP)
      WD=-ALOG(1.-RAN(KSEED))/DLAM
      UTIME=UTIME+WU
      IF(UTIME.GT.TVALUE)GO TO 83
      DTIME=DTIME+WD
      T=UTIME+DTIME
      IF(T.GT.TVALUE)GO TO 84
      GO TO 85
83     D(NIT,I)=DTIME
      GO TO 82
84     D(NIT,I)=DTIME+TVALUE-T
      GO TO 82
90     WRITE(6,91)
91     FORMAT( ' ENTER NOTHING FOR UPTIME AND THE ENDPOINTS FOR
3     DOWNTIME DISTRIBUTIONS ')
      READ(5,*)DINIT,DFINAL
      NIT=0
92     NIT=NIT+1
      IF(NIT.GT.NITER)GO TO 3
      UTIME=0.
      DTIME=0.
95     JP=INT(RAN(KSEED)*100.)+1
      WU=UP(JP)
      WD=RAN(KSEED)*(DFINAL-DINIT)+DINIT
      UTIME=UTIME+WU
      IF(UTIME.GT.TVALUE)GO TO 93
      DTIME=DTIME+WD

```

```

T=UTIME+DTIME
IF(T.GT.TVALUE)GO TO 94
GO TO 95
93 D(NIT,I)=DTIME
GO TO 92
94 D(NIT,I)=DTIME+TVALUE-T
GO TO 92
100 NIT=0
102 NIT=NIT+1
IF(NIT.GT.NITER)GO TO 3
UTIME=0.
DTIME=0.
105 JP=INT(RAN(KSEED)*100.)+1
WU=UP(JP)
JD=INT(RAN(KSEED)*100.)+1
WD=DP(JD)
UTIME=UTIME+WU
IF(UTIME.GT.TVALUE)GO TO 103
DTIME=DTIME+WD
T=UTIME+DTIME
IF(T.GT.TVALUE)GO TO 104
GO TO 105
103 D(NIT,I)=DTIME
GO TO 102
104 D(NIT,I)=DTIME+TVALUE-T
GO TO 102
3 CONTINUE
DO 205 J=1,NITER
SUM=0.
DO 206 I=1,NT
VEC(J)=SUM+D(J,I)
206 CONTINUE
205 CONTINUE
CALL SORT (VEC,NITER)
DO 207 J=1,20
K=INT(0.05*J*NITER)
EP=1.-FLOAT(J)*0.05
WRITE(6,*)VEC(K),EP
207 CONTINUE
STOP
END

```

C  
C  
C  
C

```

SUBROUTINE SORT (VEC1,NITER)
DIMENSION VEC1(1000)
DO 10 I=1,NITER
DO 20 J=I,NITER
IF(VEC1(I).GT.VEC1(J))THEN
X=VEC1(I)
VEC1(I)=VEC1(J)
VEC1(J)=X
ENDIF

```

```

20 CONTINUE
10 CONTINUE
RETURN
END

C
C
C
C
SUBROUTINE EMPDU
COMMON/ALI/UP(101),DP(101)
5 READ(1,*,END=200)PERN,VALUEN
IF(PERN.EQ.0.)THEN
PERO=PERN
VALUEO=VALUEN
READ(1,*,END=200)PERN,VALUEN
ENDIF
JPERN=INT(100*PERN)+1
JPERO=INT(100*PERO)+1
IF(JPERN.EQ.JPERO)GO TO 20
DO 10 I=JPERO,JPERN
UP(I)=((VALUEN-VALUEO)/(JPERN-JPERO))*(I-JPERO)+VALUEO
10 CONTINUE
20 PERO=PERN
VALUEO=VALUEN
GO TO 5
200 IF(PERO.EQ.1)GO TO 210
DO 205 I=JPERN,101
205 UP(I)=UP(JPERN)
210 RETURN
END

C
C
C
C
SUBROUTINE EMPDD
COMMON/ALI/UP(101),DP(101)
5 READ(2,*,END=200)PERN,VALUEN
IF(PERN.EQ.0.)THEN
PERO=PERN
VALUEO=VALUEN
READ(2,*,END=200)PERN,VALUEN
ENDIF
JPERN=INT(100*PERN)+1
JPERO=INT(100*PERO)+1
IF(JPERN.EQ.JPERO)GO TO 20
DO 10 I=JPERO,JPERN
DP(I)=((VALUEN-VALUEO)/(JPERN-JPERO))*(I-JPERO)+VALUEO
10 CONTINUE
20 PERO=PERN
VALUEO=VALUEN
GO TO 5
200 IF(PERO.EQ.1)GO TO 210
DO 205 I=JPERN,101
205 DP(I)=DP(JPERN)
210 RETURN
END

```

## SIMUL2 PROGRAM

```

p*c
PROGRAM SIMUL2
COMMON/ALI/UP(101,10)
real*8 valueo,valuen,pern,pero
CHARACTER*80 UPFILE
DIMENSION R(10),V(10),D(1000),WD(1000)
C THIS PROGRAM IS USED TO CALCULATE THE DISTRIBUTION OF
C SUMMATION OF N RANDOM VARIABLES WITH EMPIRICAL
C DISTRIBUTION. N IS TO BE LESS THAN 10 AND THE EMPIRICAL DISTRIBUTIONS
C ARE WRITTEN IN THE FORM OF THEIR PERCENTILES IN THE FORM OF
C EXCEEDENCE PROBABILITY WHICH USUALLY IS GENERATED BY ACOT.NOTE
C THERE IS A NEED FOR THE FIRST ENTRY TO BE THE ZERO PERCENTILE AND
C THE LAST ENTRY TO BE THE 100 PERCENTILE.THE OUTPUT FILE GENERATED
C WOULD BE UNDER SIMOUT.DAT.THE PROGRAM EXPECT TO SEE THE WEIGHTING
C FACTORS IN THE INPUT FILE THAT IS r(i) in r(i)*x(i)
OPEN(UNIT=2,FILE='SIMOUT',STATUS='NEW',ERR=25)
25 CONTINUE
WRITE(6,6)
6 FORMAT( ' ENTER THE NAME OF THE FILE FOR EMPIRICAL DATA ')
READ(5,7)UPFILE
7 FORMAT(A)
GO TO 3
9 STOP ' NO INPUT FILE '
3 WRITE(6,8)
8 FORMAT( ' ENTER SEED AND THE NUMBER OF VARIABLES')
READ(5,*)KSEED,N
IF(N.GT.10)STOP ' TOO MANY VARIABLES'
DO 10 J=1,N
READ(1,*,END=10)R(J)
c*****temporary write statements*****
C WRITE(6,*)R(J)
5 read(1,*)valuen,pern
c*****temporary write statement*****
C write(6,*)valuen,pern
PERN=1.-PERN
IF(PERN.EQ.0.)THEN
PERO=PERN
VALUEO=VALUEN
READ(1,*)VALUEN,PERN
PERN=1.-PERN
ENDIF
21 JPERN=INT(100*PERN)+1
JPERO=INT(100*PERO)+1
C write(6,*)JPERN,JPERO
IF(JPERN.EQ.JPERO)GO TO 20
DO 11 I=JPERO,JPERN
UP(I,J)=((VALUEN-VALUEO)/float(JPERN-JPERO))*float(I-JPERO)
up(i,j)=up(i,j)+valueo
11 CONTINUE
if(pern.eq.1.)go to 10
GO TO 101
20 if(pern.eq.1.)go to 10
READ(1,*)VALUEN,PERN
PERN=1.-PERN
GO TO 21

```

```

101  VALUEO=VALUEN
      PERO=PERN
      GO TO 5
10   CONTINUE
****temporary write statements for checking****
      write(6,*)(up(50,j),j=1,n)
      NIT=0
      DO 30 J=1,N
      DO 40 I=2,100
      DO 50 K=1,N
      IF(K.EQ.J)THEN
          V(K)=UP(I,J)
          GO TO 50
      ENDIF
      JJ=INT(RAN(KSEED)*100)+1
      V(K)=UP(JJ,K)
50   CONTINUE
      NIT=NIT+1
      D(NIT)=0.
      WD(NIT)=0.
      DO 60 K=1,N
      D(NIT)=D(NIT)+V(K)
      WD(NIT)=WD(NIT)+R(K)*V(K)
60   CONTINUE
40   CONTINUE
30   CONTINUE
      CALL SORT (D,NIT)
      CALL SORT (WD,NIT)
      DO 207 J=1,20
      K=INT(0.05*Float(J*NIT))
      EP=1.-FLOAT(J)*0.05
      if (j.eq.1) WRITE(2,70)
70   FORMAT( ' CUMULATIVE DOWNTIME DISTRIBUTION PERCENTILES' )
      WRITE (2,72)D(K),EP
72   FORMAT(1X,f9.2,1X,f9.2)
207  CONTINUE
      DO 208 J=1,20
      K=INT(0.05*J*NIT)
      EP=1.-FLOAT(J)*0.05
      if (j.eq.1) WRITE(2,80)
80   FORMAT( ' CUMULATIVE DOWNTIME RISK DISTRIBUTION PERCENTILES' )
      WRITE (2,82)WD(K),EP
82   FORMAT(1X,E9.2,1X,E9.2)
208  CONTINUE
      STOP
      END
C SUBROUTINE FOR SORTING OF A VECTOR
      SUBROUTINE SORT (VEC1,NITER)
      DIMENSION VEC1(1000)
      DO 10 I=1,NITER
      DO 20 J=I,NITER
      IF(VEC1(I).GT.VEC1(J))THEN

```

```
                X=VEC1(I)  
                VEC1(I)=VEC1(J)  
                VEC1(J)=X  
20            ENDIF  
10            CONTINUE  
            CONTINUE  
            RETURN  
            END
```

## PFM PROGRAM

```

*c
PROGRAM PLOTGEN
C
C A GENERAL PURPOSE PLOT PROGRAM - M.A. AZARM, SEPT. 14, 1968
C
C ANY NUMBER OF PLOTS CAN BE INPUT, EACH PLOT CAN HAVE
C UP TO 5 CURVES. CURVES ARE DRAWN WITH INTERRUPTED LINES.
C OR UP TO 3 CURVES CAN BE MARKED BY SYMBOLS AND UNCONNECTED
C LINES. LINE CURVE DATA MUST PRECEDE SYMBOL CURVE DATA.
C
COMMON /TEXT/ ICAP1, ICAP2, XYLAB(2), LEG(5)
CHARACTER*60 ICAP1 /' '/, ICAP2 /' '/
character*30 XYLAB /2*' '/
CHARACTER*20 LEG /5*' '/
COMMON /PLTDAT/ X(200,5), Y(200,5), ICTR(5)
C
CHARACTER*10 IX, IY, IEND
DATA IEND/'END'/
C
C OPEN INPUT, OUTPUT FILES.
GO TO 20
10 CONTINUE
STOP ' CANNOT OPEN OUTPUT FILE PLOTOUT - ABORT'
20 CONTINUE
OPEN(UNIT=5, FILE='PLOTIN', STATUS='OLD', ERR=30, READONLY)
GO TO 40
30 PRINT*, ' USER INPUT FILE PLOTIN.DAT DOES NOT EXIST '
STOP 'PLOTGEN ABORT '
40 CONTINUE
C
CALL COMPRS
C READ FIRST LINE OF CAPTION
100 READ(5, 1000, END=900) ICAP1
C PRINT*, ICAP1
110 READ (5, 1000) ICAP2
C PRINT*, ICAP2
READ (5, 1000) XYLAB(1), XYLAB(2)
C PRINT*, XYLAB(1)
C PRINT*, XYLAB(2)
READ(5, 3000) INP, LNP
C READ INP DATA SETS
DO 230 I=1, INP
READ(5, 1000) (LEG(I))
N=0
210 READ(5, 1000) IX, IY
IF(IX.EQ.IEND) GO TO 220
N=N+1
DECODE(10, 5000, IX) X(N, I)
DECODE(10, 5000, IY) Y(N, I)
GO TO 210
220 ICTR(I)=N
C PRINT*, N
230 CONTINUE
C

```

```

C      DRAW THE PLOT
      CALL PLOTIT (INP,LNP)
      GO TO 100
900   CALL DONEPL
      STOP
C
1000  FORMAT(2A)
3000  FORMAT(2I5)
5000  FORMAT(E10.3)
C
      END
      SUBROUTINE PLOTIT (INP,LNP)
      COMMON /TEXT/ ICAP1,ICAP2,XYLAB(2),LEG(5)
      CHARACTER*60 ICAP1,ICAP2
      CHARACTER*30 XYLAB
      CHARACTER*20 LEG
      INTEGER*4 IC1(15),IC2(15),XYLX(8),XYLY(8),ILEG(5)
      COMMON /PLTDAT/ X(200,5),Y(200,5),ICTR(5)
      DIMENSION XLO(5),XHI(5),YLO(5),YHI(5)
      DIMENSION IPAR(100)
      DATA XLO,YLO/10*1.E+38/, XHI,YHI/10*1.E-38/
      DATA IP/0/
      IP=IP+1
      CALL BGNPL (IP)
      CALL NOBRDR
C      FIND AXES LIMITS AND SCALING
      DO 10 I=1,INP
      N=ICTR(I)
      CALL MINMAX (X(1,I),XLO(I),XHI(I),N,1,1)
10    CALL MINMAX (Y(1,I),YLO(I),YHI(I),N,1,1)
      XMIN=AMIN1(XLO(1),XLO(2),XLO(3),XLO(4),XLO(5))
      XMAX=AMAX1(XHI(1),XHI(2),XHI(3),XHI(4),XHI(5))
      YMIN=AMIN1(YLO(1),YLO(2),YLO(3),YLO(4),YLO(5))
      YMAX=AMAX1(YHI(1),YHI(2),YHI(3),YHI(4),YHI(5))
C      PRINT*,XMIN,XMAX,YMIN,YMAX
C
      ENCODE(60,1010,IC1)ICAP1
      ENCODE(60,1010,IC2)ICAP2
      ENCODE(30,1010,XYLX)XYLAB(1)
      ENCODE(30,1010,XYLY)XYLAB(2)
      CALL HEIGHT(0.18)
      XAXIS=7.0
      YAXIS=5.5
      CALL TITLE(1H,-1,XYLX,100,XYLY,100,XAXIS,YAXIS)
      CALL FRAME
      CALL HEADIN(IC1,-100,1.10,2)
      CALL HEADIN(IC2,100,1.05,2)
      XORIG=XMIN
      YORIG=YMIN
      CALL GRAF(0.,50.,300.,YORIG,'SCALE',YMAX)
      JL=LINEST(IPAR,100,40)

```

```

CALL HEIGHT(.18)
CALL LINESP(1.75)
NS=0
DO 20 K=1,LNP
ENCODE(20,1010,ILEG)LEG(K)
CALL LINES(ILEG,IPAR,K)
IF(K.EQ. 2) CALL DASH
IF(K.EQ. 3) CALL CHNDOT
IF(K.EQ. 4) CALL DOT
IF(K.EQ. 5) CALL CHNSH
CALL LEGLIN
20 CALL CURVE(X(1,K),Y(1,K),ICTR(K),NS)
NP=INP-LNP
IF(NP.LE. 0) GO TO 40
NS=-1
DO 30 I=1,NP
IF(I.EQ. 1) CALL MARKER (4)
IF(I.EQ. 2) CALL MARKER (5)
IF(I.EQ. 3) CALL MARKER (7)
K=LNP+I
ENCODE(20,1010,ILEG)LEG(K)
CALL LINES (ILEG,IPAR,K)
CALL CURVE (X(1,K),Y(1,K),ICTR(K),NS)
30 CONTINUE
40 CONTINUE
XL=XLEGND(IPAR,INP)
YL=YLEGND(IPAR,INP)
XS=.07
YS=.07
XA=XAXIS
YA=YAXIS
X1=XA-XL-XS
Y1=YA-YL-YS
C PRINT*,XA,XL,X1
C PRINT*,YA,YL,Y1
C*****THIS IS THE STATEMENT REGARDING THE LEGENDS*****
C CALL LEGEND(IPAR,INP,X1,Y1)
CALL ENDPL (0)
RETURN
1010 FORMAT(A)
END
SUBROUTINE MINMAX (A,AMIN,AMAX,NPTS,ND,IDIM)
C
C FIND THE MINIMUM AND MAXIMUM VALUES IN ARRAY A
C
DIMENSION A(IDIM,ND)
AMIN=A(1,1)
AMAX=A(1,1)
DO 10 M=1,ND
DO 10 N=1,NPTS
IF(A(N,M).LT. AMIN) AMIN=A(N,M)
IF(A(N,M).GT. AMAX) AMAX=A(N,M)

```

```
10 CONTINUE
   RETURN
   END
*c
   PROGRAM CUMDON
C   TEST PROGRAM FOR THE VAX/VMS
C
C   DIMENSION CDT(400),CDP(400)
   DIMENSION DT(20),DTU(20),DTL(20),FDF(20,20),FDT(20,20)
   REAL*8 XMOM(8),XMO(8),VXMO(8)
   common/junk/aro,xm1o,xm2o
   COMMON F(20)
C
C   OPEN INPUT AND OUTPUT FILES
   OPEN(UNIT=6,FILE='OUTPUT',STATUS='NEW',ERR=25)
   GO TO 15
25 CONTINUE
   STOP 'CANNOT OPEN OUTPUT FILE - ABORT'
15 CONTINUE
   OPEN(UNIT=5,FILE='INPUT',STATUS='OLD',ERR=35,READ ONLY)
   GO TO 20
35 CONTINUE
   STOP 'NO INPUT FILE'
```

## SAMPLE INPUT FILE FOR PFM PROGRAM

```

*o
Figure-18: Comparison of compound poisson process with actual$
(cumulative risk including the undetected time)
CUMULATIVE RISK(core melt)$ EXCEEDENCE PROBABILITY$
  2 2
COMPOUND$
0.25E-04 0.73E+00
0.50E-04 0.53E+00
0.76E-04 0.38E+00
0.10E-03 0.27E+00
0.13E-03 0.19E+00
0.15E-03 0.13E+00
0.18E-03 0.91E-01
0.20E-03 0.62E-01
0.23E-03 0.42E-01
0.25E-03 0.28E-01
0.28E-03 0.18E-01
0.30E-03 0.11E-01
0.33E-03 0.72E-02
0.35E-03 0.44E-02
0.38E-03 0.26E-02
0.43E-03 0.82E-03
0.45E-03 0.40E-03
0.48E-03 0.15E-03
0.50E-03 0.00E+00
END
ACTUAL$
0.33E-04 0.95E+00
0.38E-04 0.90E+00
0.41E-04 0.85E+00
0.45E-04 0.80E+00
0.47E-04 0.75E+00
0.49E-04 0.70E+00
0.52E-04 0.65E+00
0.55E-04 0.60E+00
0.57E-04 0.55E+00
0.60E-04 0.50E+00
0.63E-04 0.45E+00
0.66E-04 0.40E+00
0.69E-04 0.35E+00
0.73E-04 0.30E+00
0.77E-04 0.25E+00
0.82E-04 0.20E+00
0.89E-04 0.15E+00
0.10E-03 0.10E+00
0.19E-03 0.50E-01
0.28E-03 0.00E+00
END

```

